2 odds
6 evens

8 problems $\frac{}{16}$

Differentiation with ln and e (odds)

1. $f(x) = \ln \frac{x^2}{\sqrt{x}} = 2 \ln x - \frac{1}{2} \ln x$

$$\Rightarrow \frac{2}{x} - \frac{1}{2x} = \frac{4-1}{2x} = \boxed{\frac{3}{2x}} = f'(x)$$

3. $f(x) = \ln(x + \sqrt{x}) \Rightarrow \frac{1}{x+\sqrt{x}} \cdot \left(1 + \frac{1}{2\sqrt{x}}\right)$

$$f'(x) = \boxed{\left(\frac{1}{x+\sqrt{x}}\right)\left(\frac{2\sqrt{x}+1}{2\sqrt{x}}\right)} \quad \text{or} \quad \boxed{\frac{2\sqrt{x}+1}{2x(\sqrt{x}+1)}}$$

5. $f(x) = \frac{(\ln x)^2}{\sqrt{x}} \Rightarrow \frac{\sqrt{x} \cdot 2(\ln x) \cdot \frac{1}{x} - (\ln x)^2 \cdot \frac{1}{2\sqrt{x}}}{(\sqrt{x})^2}$

$$= \frac{\frac{2(\ln x)}{\sqrt{x}} - \frac{(\ln x)^2}{2\sqrt{x}}}{x}$$

$$= \frac{4(\ln x) - (\ln x)^2}{2\sqrt{x}} \cdot \frac{1}{x}$$

$$f'(x) = \boxed{\frac{\ln x\,(4 - (\ln x))}{2x^{3/2}}}$$

7. $f(x) = \ln(x + \sqrt{x})^2 \Rightarrow \frac{1}{(x+\sqrt{x})^2} \cdot 2(x+\sqrt{x})\left(1 + \frac{1}{2\sqrt{x}}\right)$

$$= \frac{1}{(x+\sqrt{x})^2} \cdot \frac{(x+\sqrt{x})(2\sqrt{x}+1)}{\sqrt{x}}$$

$$= \boxed{\frac{1}{(x+\sqrt{x})} \cdot \frac{(2\sqrt{x}+1)}{\sqrt{x}}} \qquad \boxed{\frac{e^x(x \ln x - 1)}{x(\ln x)^2}}$$

9. $f(x) = \frac{e^x}{\ln x} \Rightarrow \frac{\ln x\, e^x - \frac{e^x}{x}}{(\ln x)^2} = \boxed{\frac{x(\ln x)e^x - e^x}{x(\ln x)^2}} = f'($

11. $f(x) = \dfrac{\ln x^2}{e^{x^2}} = \dfrac{2 \ln x}{e^{x^2}} \Rightarrow \dfrac{e^{x^2} \cdot \frac{2}{x} - 2 \ln x \, e^{x^2} \cdot 2x}{(e^{x^2})^2}$

$$= \frac{2e^{x^2}(1 - 2x^2(\ln x))}{x(e^{x^2})^2}$$

$$f'(x) = \boxed{\frac{2(1 - 2x^2(\ln x))}{x e^{x^2}}}$$

13. $f(x) = \dfrac{\ln x}{e^x + e^{-x}} \Rightarrow \dfrac{(e^x + e^{-x}) \cdot \frac{1}{x} - \ln x (e^x + e^{-x} \cdot (-1))}{(e^x + e^{-x})^2}$

$$f'(x) = \boxed{\frac{(e^x + e^{-x}) - x(\ln x)(e^x - e^{-x})}{x(e^x + e^{-x})^2}}$$

15. $f(x) = \ln e^x \Rightarrow \dfrac{1}{e^x} \cdot e^x = \boxed{1} = f'(x)$

OR

$f(x) = \ln e^x = x \ln e = x$

$f'(x) = \boxed{1}$

or $\ln e^x = x$

$\dfrac{d}{dx} x = \boxed{1}$

17. $f(x) = \ln x (e^{x^2} - e^{-2x})$

$$f'(x) = \ln x (e^{x^2} \cdot 2x + 2e^{-2x}) + \frac{e^{x^2} - e^{-2x}}{x}$$

$$= \frac{x(\ln x)(2x e^{x^2} + 2e^{-2x}) + (e^{x^2} - e^{-2x})}{x}$$

19. $f(x) = e^{-x}(\ln x)$

$f'(x) = e^{-x} \cdot \frac{1}{x} + (-e^{-x})(\ln x)$

$= \boxed{e^{-x}\left(\frac{1 - x(\ln x)}{x}\right)}$ or $\boxed{\frac{1 - x\ln x}{xe^{x}}}$

21. $\int \frac{e^{-\frac{2}{x^3}}}{x^4}\,dx$ let $u = -2x^{-3}$

$du = 6x^{-4}\,dx$

$= \int e^{-\frac{2}{x^3}} \cdot x^{-4}\,dx$

$= \frac{1}{6}\int e^{u}\,du = -\frac{e^{u}}{6} + C = \boxed{\frac{e^{-\frac{2}{x^3}}}{6} + C}$

23. $\int \frac{e^{3x} - e^{2x}}{e^{-x}}\,dx = \int \frac{e^{3x}}{e^{-x}} - \frac{e^{2x}}{e^{-x}} = \int e^{4x} - e^{3x}\,dx$

$= \boxed{\left(\frac{e^{4x}}{4} - \frac{e^{3x}}{3}\right) + C}$

25. $\int \ln(e^{2x-1})$

$= \int 2x - 1 = \boxed{x^2 - x + C}$

Differentiation with ln and e (evens)

2. $f(x) = \frac{\ln x^2}{\sqrt{x}} = \frac{2 \ln x}{\sqrt{x}}$

$f'(x) = \sqrt{x} \cdot \frac{2}{x} - 2 \ln x \cdot \frac{1}{2\sqrt{x}}$

$= \boxed{\frac{2 - \ln x}{x\sqrt{x}}}$

(4) $f(x) = \sqrt{x} \cdot \ln x^2 = \sqrt{x} \cdot 2 \ln x$

$f'(x) = \sqrt{x} \cdot \frac{2}{x} + 2 \ln x \cdot \frac{1}{2\sqrt{x}}$

(2)

$= \boxed{\frac{2 + \ln x}{\sqrt{x}}}$ (+2)

6. $f(x) = \ln (x^2 + \sqrt{x})$

$f'(x) = \frac{1}{x^2+\sqrt{x}} \cdot \left(2x + \frac{1}{2\sqrt{x}}\right) = \boxed{\frac{1}{x^2+\sqrt{x}} \left(\frac{4x^{3/2}+1}{2\sqrt{x}}\right)}$

(2) 8. $f(x) = \frac{\ln x}{e^x}$

(+1) (+2)

$f'(x) = \frac{\frac{e^x}{x} - \ln x \, e^x}{(e^x)^2} = \frac{e^x - xe^x \ln x}{xe^{2x}} = \boxed{\frac{1 - x\ln x}{xe^x}}$

10. $f(x) = \frac{\ln x^2}{e^{2x}} = \frac{2 \ln x}{e^{2x}}$

$f'(x) = \frac{e^{2x} \cdot \frac{2}{x} - 2 \ln x \cdot e^{2x} \cdot 2}{(e^{2x})^2}$

$= \frac{2e^{2x} - 4x \ln x e^{2x}}{(e^{2x})^2} = \boxed{\frac{2 - 4x \ln x}{xe^{2x}}}$

12. $f(x) = \dfrac{(\ln x)^2}{e^{3x}}$

$$f'(x) = \frac{e^{3x} \cdot 2(\ln x) \cdot \frac{1}{x} - (\ln x)^2 e^{3x} \cdot 3}{(e^{3x})^2}$$

$$= \frac{2(\ln x) - 3x(\ln x)^2}{x e^{3x}} = \boxed{\ln x\left(\frac{2 - 3x(\ln x)}{x e^{3x}}\right)}$$

14. $f(x) = (\ln x) e^{-x}$

(+1)

(2) $f'(x) = \ln x \cdot -e^{-x} + \dfrac{e^{-x}}{x}$

(+1)

$$= \frac{-x e^{-x} \ln x + e^{-x}}{x} = \boxed{\frac{-e^{-x}(x \ln x - 1)}{x}}$$

or

$\boxed{\dfrac{1 - x \ln x}{x e^x}}$

16. $f(x) = (\ln x) e^2 - e^{2x}$

(2) $f'(x) = \dfrac{e^2}{x} - 2e^{2x} = \boxed{\dfrac{e^2 - 2x e^{2x}}{x}}$

(+1) (+1)

18. $f(x) = e^5 \ln x$

(2) $f'(x) = \dfrac{e^5}{x}$ (+2)

20. $f(x) = \ln(\ln(e^{-2x^2}))$

$$f'(x) = \frac{1}{\ln(e^{-2x^2})} \cdot \frac{1}{e^{-2x^2}} \cdot e^{-2x^2} \cdot (-4x)$$

(2)

(+2)

$$= \frac{-4x \cancel{e^{-2x^2}}}{\cancel{e^{-2x^2}} \cdot \ln(e^{-2x^2})} = \frac{-4x}{-2x^2} = \frac{2}{x}$$

22. $\int 2x e^{-\frac{x^2}{2}} \, dx$ let $u = -\frac{x^2}{2}$

$du = -x \, dx$

$-2\int e^u \, du = -2e^u + C$

$= -2e^{-\frac{x^2}{2}} + C$

24. $\int \frac{2e^x - 2e^{-x}}{(e^x + e^{-x})^2} \, dx$ let $u = e^x + e^{-x}$

$du = e^x - e^{-x} \, dx$

$\int \frac{2(e^x - e^{-x}) \, dx}{u^2}$

$\int \frac{2}{u^2} \, du = \int 2u^{-2} \, du$

$= -2u^{-1} + C$

$= -2(e^x + e^{-x})^{-1} + C$

$= \frac{-2}{e^x + e^{-x}} + C$

26. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx$ let $u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}} \, dx$

$2\int e^u \, du = 2e^u + C$

$= 2e^{\sqrt{x}} + C$