

Integration by Substitution

Date _____ Pe

Evaluate each indefinite integral. Use the provided substitution.

$$1) \int \frac{20x^4}{4x^5 + 3} dx; u = 4x^5 + 3$$
$$\ln |4x^5 + 3| + C$$

$$2) \int 36x^2 e^{4x^3 + 3} dx; u = 4x^3 + 3$$
$$3e^{4x^3 + 3} + C$$

$$3) \int 80x^3 \cdot 3^{5x^4 - 2} dx; u = 5x^4 - 2$$
$$\frac{4 \cdot 3^{5x^4 - 2}}{\ln 3} + C$$

$$4) \int \frac{2}{x(-1 + \ln 4x)} dx; u = -1 + \ln 4x$$
$$2 \ln |-1 + \ln 4x| + C$$

Evaluate each indefinite integral.

$$5) \int \frac{12x^2}{x^3 + 2} dx$$
$$4 \ln |2x^3 + 4| + C$$

$$6) \int \frac{20e^{5x}}{e^{5x} + 3} dx$$
$$4 \ln |e^{5x} + 3| + C$$

$$7) \int 10 \sin -2x \cdot e^{\cos -2x} dx$$
$$5e^{\cos -2x} + C$$

$$8) \int \frac{5e^{-3 + \ln 3x}}{x} dx$$
$$5e^{-3 + \ln 3x} + C$$

Differentiation - Natural Logs and Exponentials

Date _____ Period _____

Differentiate each function with respect to x .

1) $y = \ln x^3$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{x^3} \cdot 3x^2 \\ &= \frac{3}{x} \end{aligned}$$

2) $y = e^{2x^2}$

$$\frac{dy}{dx} = e^{2x^2} \cdot 6x^2$$

3) $y = \ln \ln 2x^4$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{\ln 2x^4} \cdot \frac{1}{2x^4} \cdot 8x^3 \\ &= \frac{4}{x \ln 2x^4} \end{aligned}$$

4) $y = \ln \ln 3x^3$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{\ln 3x^3} \cdot \frac{1}{3x^3} \cdot 9x^2 \\ &= \frac{3}{x \ln 3x^3} \end{aligned}$$

5) $y = \cos \ln 4x^3$

$$\begin{aligned} \frac{dy}{dx} &= -\sin \ln 4x^3 \cdot \frac{1}{4x^3} \cdot 12x^2 \\ &= -\frac{3 \sin \ln 4x^3}{x} \end{aligned}$$

6) $y = e^{e^{3x^2}}$

$$\begin{aligned} \frac{dy}{dx} &= e^{e^{3x^2}} e^{3x^2} \cdot 6x \\ &= 6xe^{e^{3x^2} + 3x^2} \end{aligned}$$

7) $y = e^{(4x^3+5)^2}$

$$\begin{aligned} \frac{dy}{dx} &= e^{(4x^3+5)^2} \cdot 2(4x^3+5) \cdot 12x^2 \\ &= 24x^2 e^{(4x^3+5)^2} (4x^3+5) \end{aligned}$$

8) $y = \ln 4x^2 \cdot (-x^3 - 4)$

$$\begin{aligned} \frac{dy}{dx} &= \ln 4x^2 \cdot -3x^2 + (-x^3 - 4) \cdot \frac{1}{4x^2} \cdot 8x \\ &= \frac{-3x^3 \ln 4x^2 - 2x^3 - 8}{x} \end{aligned}$$

9) $y = \ln \left(-\frac{4x^4}{x^3-3} \right)^5$

$$\begin{aligned} \frac{dy}{dx} &= 5 \left(\frac{1}{-4x^4} \cdot -16x^3 - \frac{1}{x^3-3} \cdot 3x^2 \right) \\ &= \frac{5(x^3-12)}{x(x^3-3)} \text{ (Rules of logarithms used)} \end{aligned}$$

10) $y = \frac{e^{5x^4}}{e^{4x^2+3}}$

$$\begin{aligned} \frac{dy}{dx} &= e^{5x^4 - (4x^2+3)} (20x^3 - 8x) \\ &= 4xe^{5x^4 - 4x^2 - 3} (5x^2 - 2) \text{ (Rules of exponents used)} \end{aligned}$$