

Worksheet # 24: Definite Integrals and The Fundamental Theorem of Calculus

1. Suppose $\int_0^1 f(x) dx = 2$, $\int_1^2 f(x) dx = 3$, $\int_0^1 g(x) dx = -1$, and $\int_0^2 g(x) dx = 4$.

Compute the following using the properties of definite integrals:

(a) $\int_1^2 g(x) dx$

(b) $\int_0^2 [2f(x) - 3g(x)] dx$

(c) $\int_1^1 g(x) dx$

(d) $\int_1^2 f(x) dx + \int_2^0 g(x) dx$

(e) $\int_0^2 f(x) dx + \int_2^1 g(x) dx$

2. Simplify $\int_0^2 3f(x) dx + \int_1^3 3f(x) dx - \int_0^3 2f(x) dx - \int_1^2 3f(x) dx$

3. Find $\int_0^5 f(x) dx$ where $f(x) = \begin{cases} 3 & \text{if } x < 3 \\ x & \text{if } x \geq 3 \end{cases}$.

4. (a) State both parts of the Fundamental Theorem of Calculus using complete sentences.

(b) Find the derivative of the function $g(x) = \int_1^{x^3} \sqrt{t^5 - 1} dt$ on $(1, \infty)$.

5. Use Part I of the Fundamental Theorem of Calculus to find the derivative of the following functions:

(a) $g(x) = \int_1^x (2 + t^4)^5 dt$

(b) $F(x) = \int_x^4 \cos(t^5) dt$

(c) $h(x) = \int_0^{x^2} \sqrt[3]{1 + r^3} dr$

(d) $y(x) = \int_{\frac{1}{x^2}}^0 \sin^3(t) dt$

(e) $G(x) = \int_{\sqrt{x}}^{x^2} \sqrt{t} \sin(t) dt$

6. Use Part II of the Fundamental Theorem of Calculus to evaluate the following integrals or explain why the theorem does not apply:

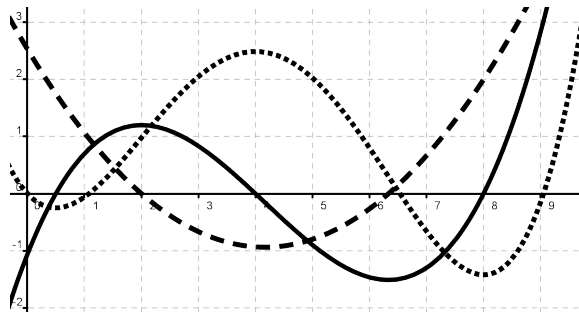
(a) $\int_{-2}^5 6x dx$

(b) $\int_{-2}^7 \frac{1}{x^5} dx$

(c) $\int_0^{\frac{\pi}{4}} \sec^2(t) dt$

(d) $\int_{\frac{\pi}{3}}^{\frac{\pi}{6}} \sin(x) dx$

7. Below is pictured the graph of the function $f(x)$, its derivative $f'(x)$, and an antiderivative $\int f(x) dx$. Identify $f(x)$, $f'(x)$ and $\int f(x) dx$.



8. Evaluate the following limits by first recognizing the sum as a Riemann sum:

(a) $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{i^3}{n^4}$

(b) $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{\sqrt{3 + \frac{i}{n}}}{n}$

(c) $\lim_{n \rightarrow \infty} \sum_{i=1}^n 2 \frac{(2 + \frac{2i}{n})^2}{n}$