monday 1/28/2019

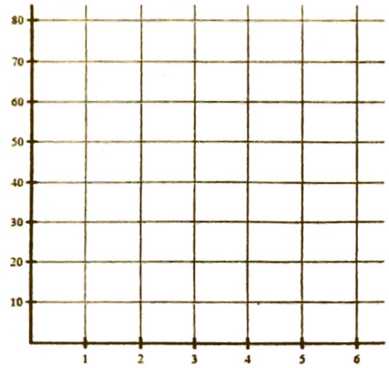
7.1 Rectangular Approximation

NOTES

CALCULUS

Write your questions here!

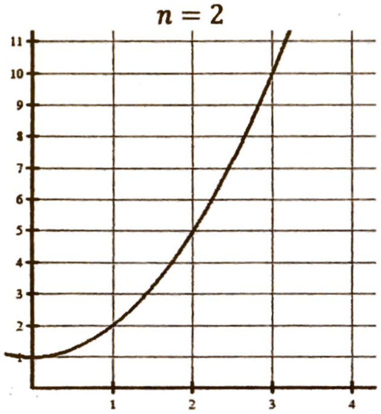
Car travels 60 miles per hour



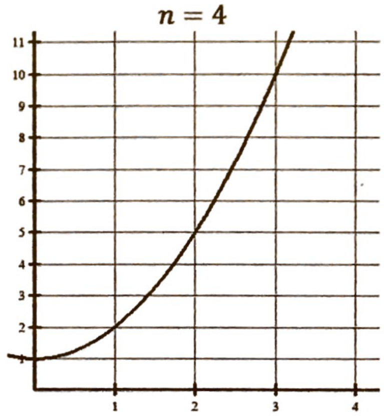
Left Endpoint Rectangle for interval [1,3] with n subintervals

$f(x) = x^2 + 1$

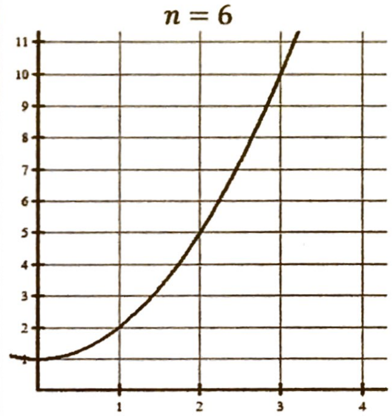
$n = 2$



$n = 4$



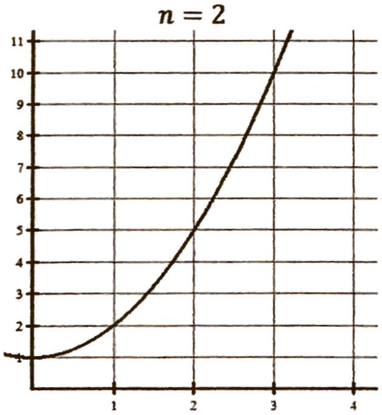
$n = 6$



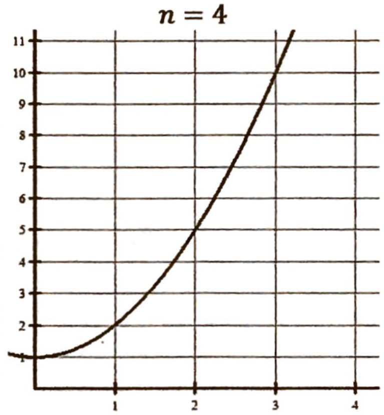
Right Endpoint Rectangle for interval [1,3] with n subintervals

$f(x) = x^2 + 1$

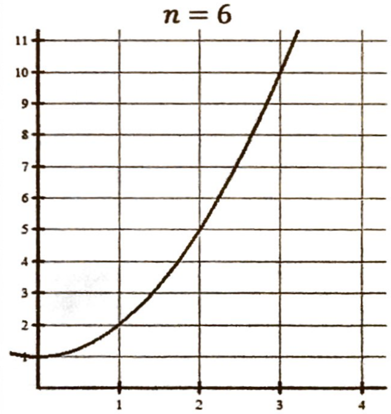
$n = 2$



$n = 4$



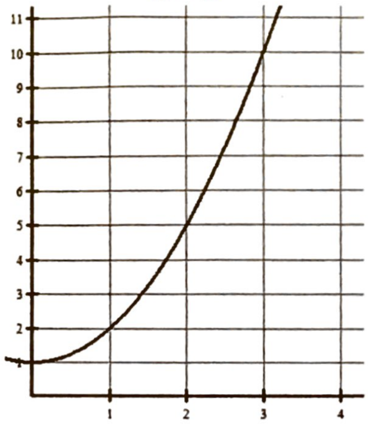
$n = 6$



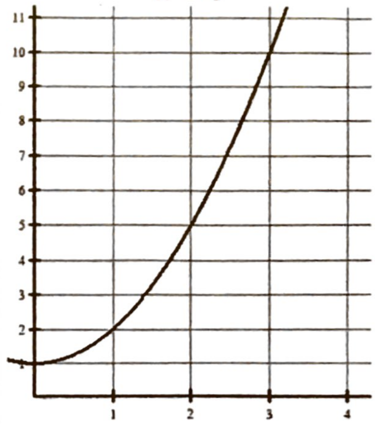
Midpoint Rectangle for interval [1,3] with *n* subintervals

$f(x) = x^2 + 1$

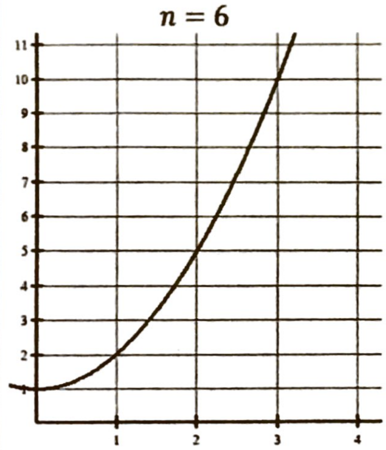
$n = 2$



$n = 4$

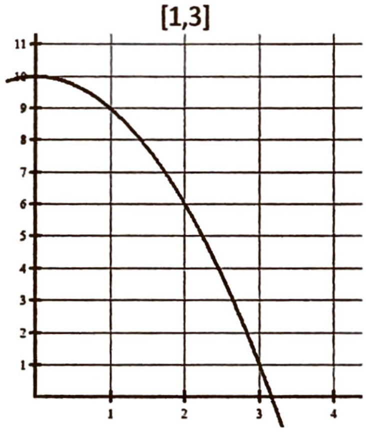


$n = 6$

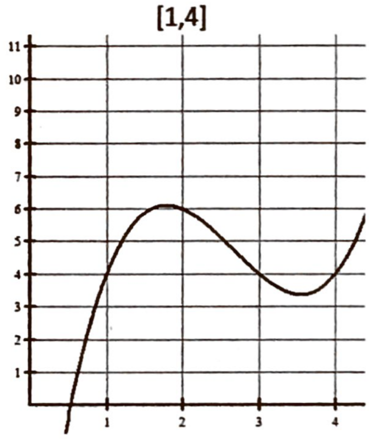


Sketch the following rectangular approximations

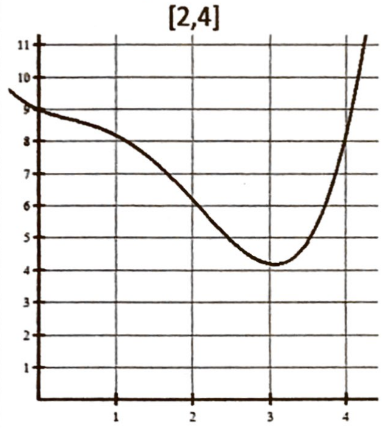
Left Endpoint $n = 5$
[1,3]



Right Endpoint $n = 8$
[1,4]



Midpoint $n = 6$
[2,4]



The rate at which water is being pumped into a tank is given by the continuous and increasing function $R(t)$. A table of selected values of $R(t)$, for the time interval $0 < t < 13$ minutes, is given below.

Time (minutes)	0	4	6	10	13
$R(t)$ (gallons/min)	7	13	18	23	27

Use right Riemann Sum with 4 subintervals to approximate the area under the curve.

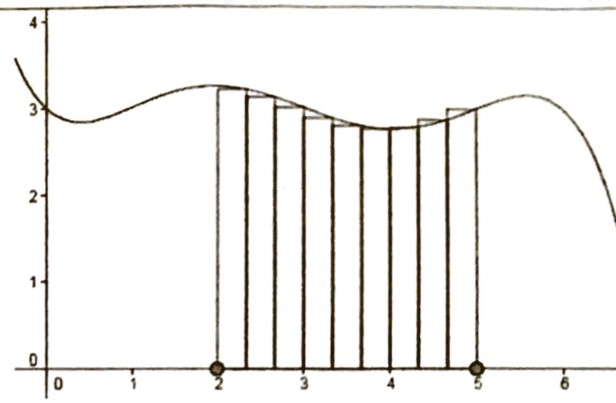
What does this represent?

Is the approximation greater or less than the true value?

SUMMARY

Now, summarize your notes here!

Use the graph to answer 1-3.



1. Is the rectangular approximation shown to the right a left endpoint, right endpoint, or midpoint approximation?

2. Is the approximation less than or greater than the true value?

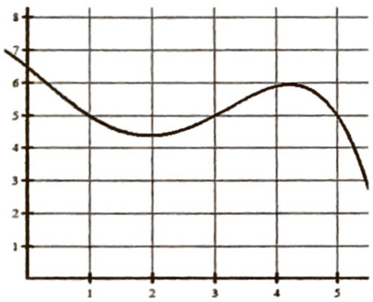
3. What is the width of each rectangle?

 You can use a calculator on 4-13 

Sketch the following rectangular approximations. Find the width of each subinterval.

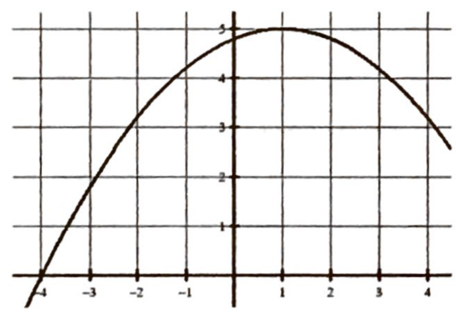
4. Midpoint on the interval [1,4] with $n = 6$ subintervals

Width of each subinterval =



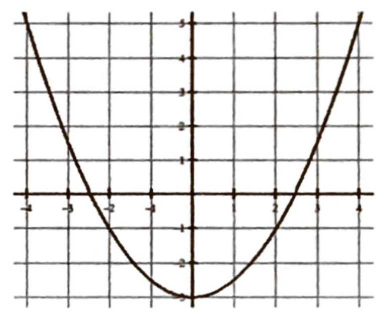
5. Right Endpoint on [-2,2] with $n = 5$ subintervals

Width of each subinterval =



6. Left Endpoint on [-2,4] with $n = 12$ subintervals

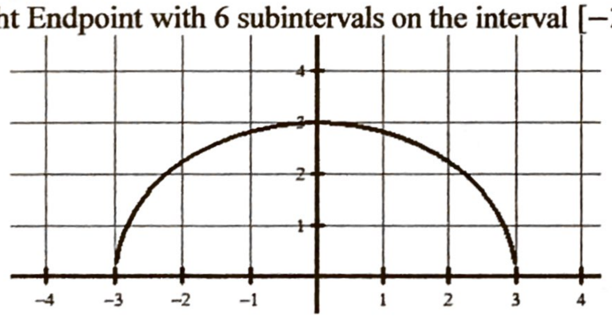
Width of each subinterval =



Approximate the area under the curve using the given rectangular approximation. Include a sketch! Justify!

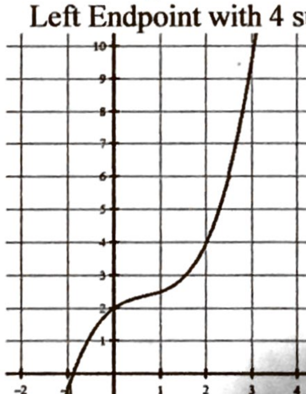
7. $f(x) = \sqrt{9 - x^2}$

Right Endpoint with 6 subintervals on the interval $[-2,1]$



8. $f(x) = \frac{1}{2}x^3 - x^2 + x + 2$

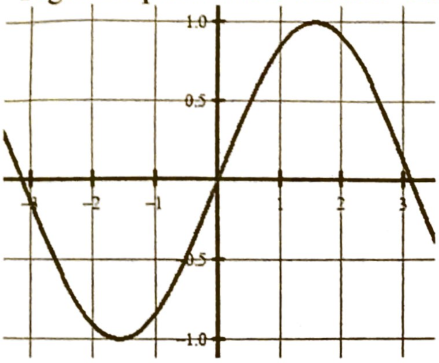
Left Endpoint with 4 subintervals on the interval [1,3]



Approximate the area under the curve using the given rectangular approximation. Include a sketch! Justify!

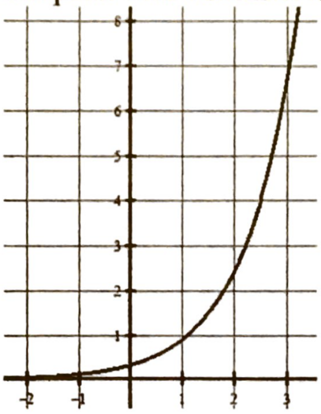
9. $f(x) = \sin x$

Right Endpoint with 3 subintervals on the interval $[0,2]$



10. $f(x) = \frac{e^x}{3}$

Midpoint with 4 subintervals on the interval $[1,3]$



Use the information provided to answer the following.

11. Let $y(t)$ represent the rate of change of the population of a town over a 20-year period, where y is a differentiable function of t. The table shows the population change in people per year recorded at selected times.

Time (years)	0	4	10	13	20
$y(t)$ (people per year)	2500	2724	3108	3697	4283

a. Use the data from the table and a right Riemann Sum with four subintervals to approximate the area under the curve.

b. What does your answer from part (a) represent?

c. Assuming that $y(t)$ is a continuous increasing function, is your approximation from part (a) greater or less than the true value?

KEY

7.1 Rectangular Approximation

PRACTICE

Use the graph to answer 1-3.

1. Is the rectangular approximation shown to the right a left endpoint, right endpoint, or midpoint approximation?

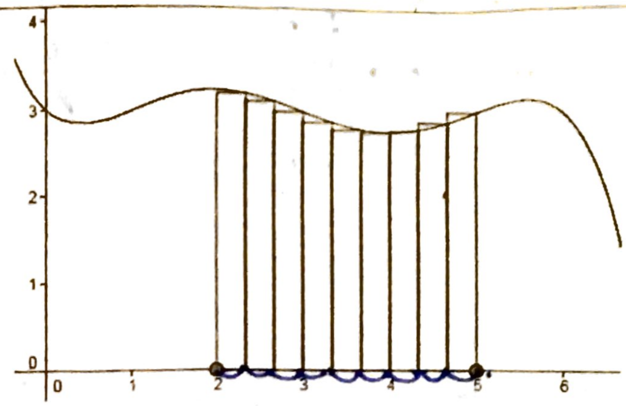
   Right Endpoint

2. Is the approximation less than or greater than the true value?

   Less than

3. What is the width of each rectangle?

   $n = 9$ $\quad \Delta x = \frac{b-a}{n} = \frac{5-2}{9} = \frac{3}{9} = \frac{1}{3}$

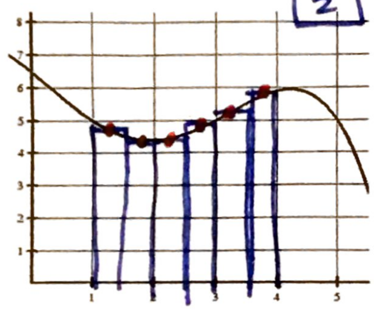


You can use a calculator on 4-13

Sketch the following rectangular approximations. Find the width of each subinterval.

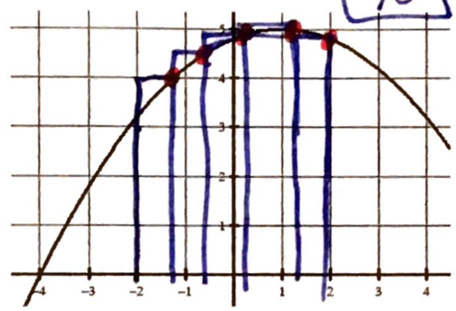
4. Midpoint on the interval [1,4] with $n = 6$ subintervals $\frac{4-1}{6} = \frac{3}{6}$

   Width of each subinterval = $\frac{1}{2}$



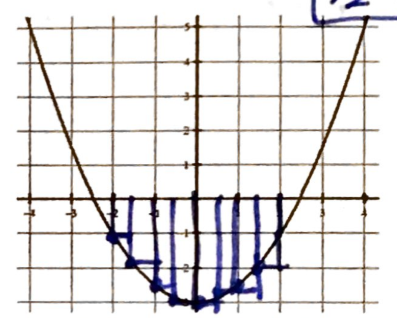
5. Right Endpoint on [-2,2] with $n = 5$ subintervals $\frac{2--2}{5} =$

   Width of each subinterval = 4/5



6. Left Endpoint on [-2,4] with $n = 12$ subintervals $\frac{4--2}{12} = \frac{6}{12}$

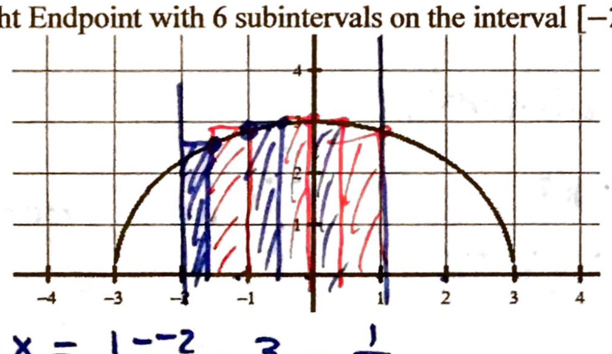
   Width of each subinterval = ½



Approximate the area under the curve using the given rectangular approximation. Include a sketch! Justify!

7. $f(x) = \sqrt{9 - x^2}$

   Right Endpoint with 6 subintervals on the interval $[-2,1]$



$\Delta x = \frac{1--2}{6} = \frac{3}{6} = \frac{1}{2}$

Setup

$\frac{1}{2}[f(-2+\frac{1}{2}) + f(-2+1) + f(-2+1.5) + f(-2+2) + f(-2+2.5) + f(1)]$

−1.5

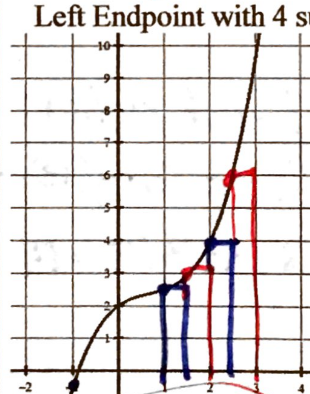
$\frac{1}{2}[f(-1.5) + f(-1) + f(-.5) + f(0) + f(\frac{1}{2}) + f(1)]$

8.585505122 Answer

8.586

8. $f(x) = \frac{1}{2}x^3 - x^2 + x + 2$

   Left Endpoint with 4 subintervals on the interval [1,3]



$\Delta x = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2}$

$\frac{1}{2}[f(1) + f(1.5) + f(2) + f(2.5)]$

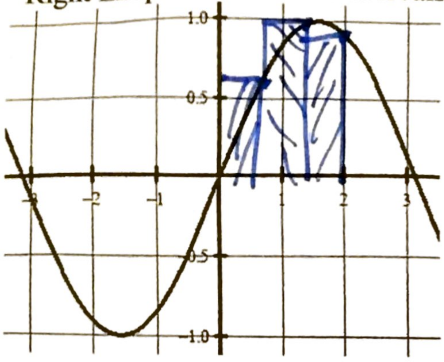
~~4.125~~

7.75

Approximate the area under the curve using the given rectangular approximation. Include a sketch! Justify!

9. $f(x) = \sin x$

Right Endpoint with 3 subintervals on the interval [0,2]



$\frac{2-0}{3} = \frac{2}{3}$

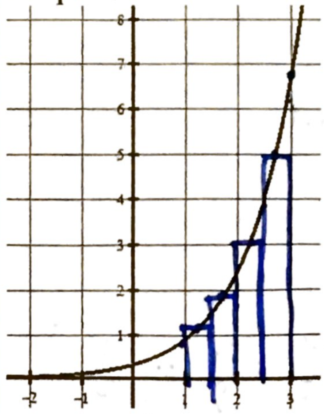
$\frac{2}{3}[f(2/3) + f(4/3) + f(2)]$

$\frac{2}{3}[\sin\frac{2}{3} + \sin 4/3 + \sin 2]$

About 1.666

10. $f(x) = \frac{e^x}{3}$

Midpoint with 4 subintervals on the interval [1,3]



$\frac{3-1}{4} = \frac{2}{4} = \frac{1}{2}$

$\frac{1}{2}[f(1.25) + f(1.75) + f(2.25) + f(2.75)]$

5.729

Use the information provided to answer the following.

11. Let $y(t)$ represent the rate of change of the population of a town over a 20-year period, where y is a differentiable function of t. The table shows the population change in people per year recorded at selected times.

Time (years)	0	4	10	13	20
$y(t)$ (people per year)	2500	2724	3108	3697	4283

a. Use the data from the table and a right Riemann Sum with four subintervals to approximate the area under the curve.

Note: The width of each rectangle is different!

$4 \cdot y(4) + 6(y(10)) + 3 \cdot y(13) + 7 \cdot y(20)$

$4(2724) + 6(3108) + 3(3697) + 7(4283)$

70616 people

b. What does your answer from part (a) represent?

Total population of the town after 20 years.

c. Assuming that $y(t)$ is a continuous increasing function, is your approximation from part (a) greater or less than the true value?

over estimate – greater than true value