$\boxed{\text{AP Calculus}}$ – Riemann Sums Worksheet

Name:____________________

A Graphing Calculator is allowed for these problems.

For problem #1 and #2, fill in the table of four subdivisions to find the definite integral using the:

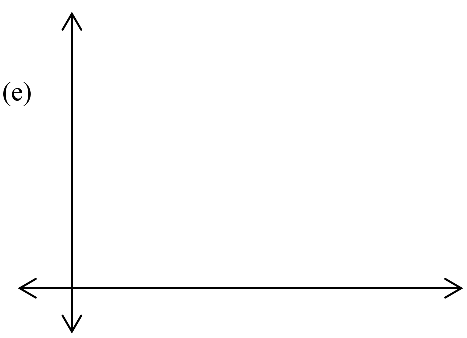
(a) Left-hand Sum, (b) Right-hand Sum,
(c) Trapezoid Rule, (d) Calculator's MATH 9 function, and then
(e) Sketch the graph of $f(x)$ and the rectangles created by the Left- and Right- Hand sums.

1. $\int_0^2 x^3\,dx$ (a) $L_f =$ (b) $R_f =$

x					
$f(x)$					

(c) $T_f =$ (d) Actual =

(e)

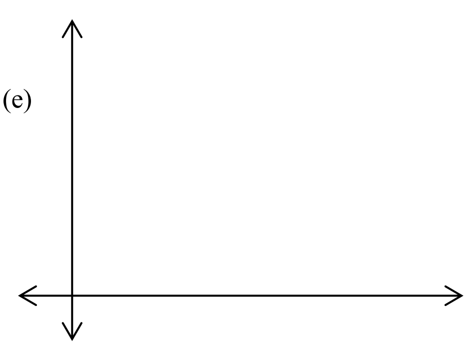


2. $\int_0^{\frac{\pi}{2}} \cos x\,dx$ (a) $L_f =$ (b) $R_f =$

x					
$f(x)$					

(c) $T_f =$ (d) Actual =

(e)



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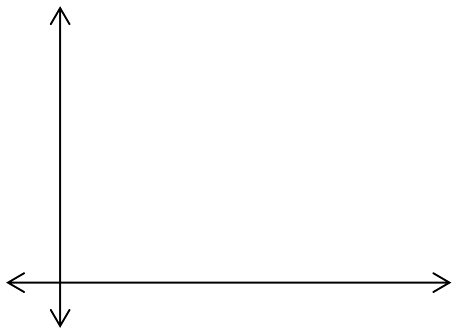
3. (a) Use four subdivisions and the Midpoint Rule to evaluate $\int_1^2 e^x \, dx$.

x				
$f(x)$				

(b) Sketch the graph and identify graphically the value of the definite integral using the Midpoint Rule.



4. Use four subdivisions and the table on the right to find the area under $f(x)$ using (a) Trapezoid Rule and (b) Midpoint Rule.

x	0	.5	1	1.5	2	2.5	3	3.5	4
$f(x)$	0	0.3	0.8	1.7	3.5	4.0	3.8	2.6	1.2

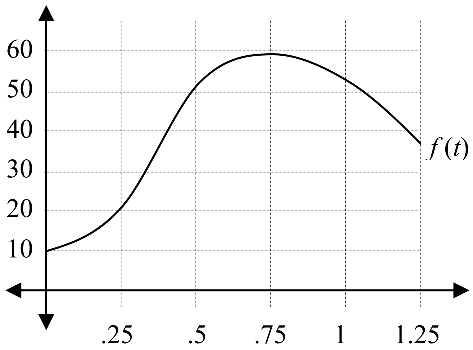
(a) $T_f =$

(b) $M_f =$

5. Consider a monotonic function over a given interval. Based on the description of each function, determine if each sum is <u>equal</u>, an <u>underestimate</u>, or an <u>overestimate</u> of the actual value of the definite integral.

DESCRIPTION	L_f	R_f	T_f	M_f
Increasing, Linear				
Decreasing, Linear				
Increasing, Concave down				
Decreasing, Concave down				
Increasing, Concave up				
Decreasing, Concave up				

6. a) Use the table below to approximate $\int_0^{1.25} f(t)\,dt$ using the Trapezoid Rule and 5 subdivisions.



b) Suppose $f(t)$ represents the velocity of a car, measured in miles per hour. Explain the practical meaning of the integral in part (a).

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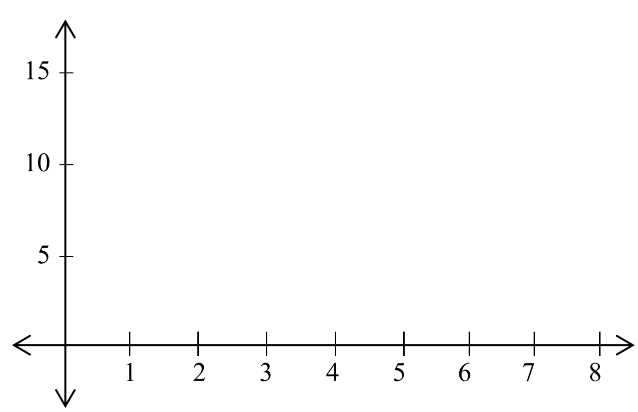
7. The table on the right represents selected values of $f(x)$.

(Note: Δx is not constant.)

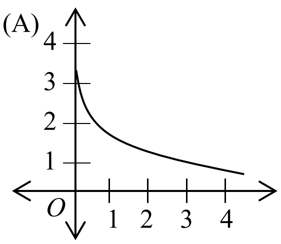
x	0	2	3	6	8
$f(x)$	1	2.5	6.75	10.5	16

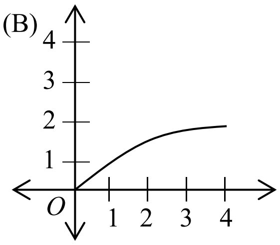
a) Find $\int_0^8 f(x)\,dx$ using the Trapezoid Rule and four subdivisions.

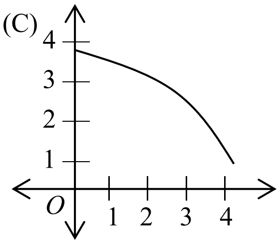
b) Sketch the four trapezoids that represents the four areas used for the trapezoid rule.



8. If a trapezoidal sum underapproximates $\int_0^4 f(x)\,dx$, and a right Riemann sum also underapproximates $\int_0^4 f(x)\,dx$, which of the following could be the graph of $y = f(x)$?

(A)


(B)


(C)


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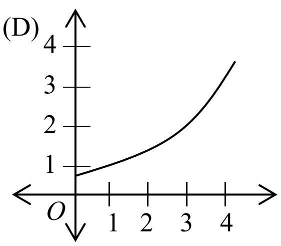
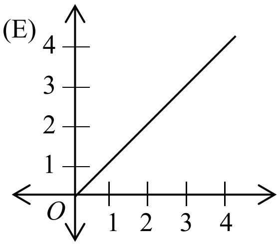
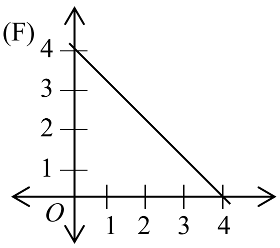


(E)


(F)


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9. Use the same choices as problem #8. If a trapezoidal sum is equal to $\int_0^4 f(x)\,dx$, and a left Riemann sum underapproximates $\int_0^4 f(x)\,dx$, which of the following could be the graph of $y = f(x)$?

ANSWERS:

1a) 2.250
b) 6.250
c) 4.250
d) 4
e) graph

2a) 1.183
b) 0.791
c) 0.987
d) 1
e) graph

3a) 4.659
b) graph
4a) 8.7
b) 8.6

5)

L_f	R_f	T_f	M_f
under	over	equal	equal
over	under	equal	equal
under	over	under	over
over	under	under	over
under	over	over	under
over	under	over	under

6a) $T = 51.625$, $M = 52.75$
b) total distance traveled
7a) $T = 60.5$
8) C
9) E