4.5 WS 1 $\qquad F'$

1. $\frac{d}{dx}\int_2^x t^2\,dt = x^2$

2. $\frac{d}{dx}\int_\pi^x \cos t\,dt = \cos x$

3. $\frac{d}{dx}\int_x^e \tan^{-1} t\,dt = \frac{d}{dx} - \int_e^x \tan^{-1} t\,dt = -\tan^{-1} x$

4. $\frac{d}{dx}\int_x^{\sqrt{2}} \frac{1}{1+t^2}\,dt = \frac{d}{dx} - \int_{\sqrt{2}}^x \frac{1}{1+t^2}\,dt = -\frac{1}{1+x^2}$

5. $\frac{d}{dx}\int_{\pi^2}^{x^2} \frac{1}{\sqrt{1-t^2}}\,dt = \frac{1}{\sqrt{1-(x^2)^2}} \cdot 2x = \frac{2x}{\sqrt{1-x^4}}$

6. $\frac{d}{dx}\int_{x^2}^{x^4} \cos t\,dt = \frac{d}{dx}\int_0^{x^4} \cos t\,dt + \frac{d}{dx}\int_{x^2}^0 \cos t\,dt$

$= \cos x^4 \cdot 4x^3 - \cos x^2 \cdot 2x$

$= 4x^3 \cos x^4 - 2x \cos x^2$

7. $\frac{d}{dx}\int_x^x \sec t\,dt = 0$

8. $\frac{d}{dx}\int_{\sin x}^{x^2} t^3\,dt = \frac{d}{dx}\int_0^{x^2} t^3\,dt + \frac{d}{dx}\int_{\sin x}^0 t^3\,dt$

$= (x^2)^3 \cdot 2x - \sin^3 x \cdot \cos x$

$= 2x^7 - \cos x \sin^3 x$

9. $\int \frac{2x}{\sqrt{x^2-1}}\,dx$ $\quad$ let $u = x^2 - 1$ $\quad du = 2x\,dx$

$\int \frac{1}{u^{1/2}}\,du = \int u^{-1/2}\,du = 2u^{1/2} + C$

$= 2(x^2-1)^{1/2} + C$

10. $\int \sec x \tan x \, dx = \sec x + C$

11. $\int (\sqrt{x} - \sqrt[5]{x})\,dx = \int (x^{1/2} - x^{1/5})\,dx = \frac{2x^{3/2}}{3} - \frac{5x^{6/5}}{6} + C$

12. $\int \frac{x^2 - 2}{x^{1/8}}\,dx = \int \left(\frac{x^2}{x^{1/8}} - 2x^{-1/8}\right)dx = \int x^{15/8} - 2x^{-1/8}$

$= \frac{8x^{23/8}}{23} - \frac{16x^{7/8}}{7} + C$

13 ln skip

14. $\int \frac{2x}{\sqrt{1+x}}\,du$ let $u = 1+x$ $du = dx$

$\int \frac{2x}{\sqrt{u}}\,du$ $x = u-1$

$\int \frac{2u-2}{u^{1/2}}\,du = \int (2u^{1/2} - 2u^{-1/2})\,du$

$= \frac{4}{3}u^{3/2} - 4u^{1/2} + C$

$= \frac{4}{3}(1+x)^{3/2} - 4(1+x)^{1/2} + C$

15. $\int \sin 3x \, dx$ let $u = 3x$ $du = 3\,dx$

$\int \sin u \frac{du}{3} = \frac{1}{3}\int \sin u \, du$ $\frac{du}{3} = dx$

$= \frac{1}{3} - \cos u$

$= -\frac{1}{3}\cos 3x + C$

16. $\int x^2 \sqrt{1-x^3}\, dx$ let $u = 1-x^3$ $du = -3x^2 dx$

$\frac{du}{-3} = x^2 dx$

$-\frac{1}{3}\int \sqrt{u}\, du = -\frac{1}{3} \cdot \frac{2}{3} u^{3/2} + C$

$= -\frac{2}{9}(1-x^3)^{3/2} + C$

17. $-\cos x \Big|_{\pi/6}^{\pi/3} = -\cos \pi/3 - -\cos \pi/6 = -\frac{1}{2} - \frac{-\sqrt{3}}{2} = \frac{-1+\sqrt{3}}{2}$

18. $\int_0^1 \frac{1}{x^2+6x+9} dx = \int_0^1 \frac{1}{(x+3)^2} dx = \int_0^1 (x+3)^{-2} dx$

let $u = (x+3)$ $du = dx$

$\int u^{-2} du = -u^{-1} = \frac{-1}{x+3}\Big|_0^1 = -\frac{1}{4} - -\frac{1}{3}$

Alternative

$\int_3^4 u^{-2} du$ $-\frac{1}{u}\Big|_3^4$

$-\frac{1}{4} - -\frac{1}{3}$

$= -\frac{3}{12} + \frac{4}{12} = \frac{1}{12}$

19 ln skip

20 ln skip

21. $\int_0^1 \frac{8}{\sqrt{3+4x}} dx$ let $u = 3+4x$ $du = 4dx$

$2du = 8dx$

$\int \frac{2du}{u^{1/2}} = 2\int u^{-1/2} du$

$= 2 \cdot 2u^{1/2} = 4(3+4x)^{1/2}\Big|_0^1$

$= 4\sqrt{7} - 4\sqrt{3}$

22. $\int_0^{\pi} \sin 3x \, dx = -\frac{1}{3}\cos 3x \Big|_0^{\pi} = -\frac{1}{3}\cos 3\pi - -\frac{1}{3}\cos 0$

$= \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

23. $\int_1^3 \frac{2}{(x+1)^2}\, dx$ let $u = x+1$ $du = dx$

$u(3) = 4$

$\int_2^4 2u^{-2}\, du$ $u(1) = 2$

$2(-u^{-1}) = -\frac{2}{u}\Big|_2^4 = -\frac{1}{2} - -1 = \frac{1}{2}$

24. $\int_{\pi/4}^{\pi/2} 1 + \sin x \, dx = x - \cos x \Big|_{\pi/4}^{\pi/2}$

$= \pi/2 - \cos \pi/2 - (\pi/4 - \cos \pi/4)$

$= \frac{\pi}{2} - 0 - (\pi/4 - \frac{\sqrt{2}}{2})$

$= \pi/2 - \pi/4 + \frac{\sqrt{2}}{2}$

$= \pi/4 + \frac{\sqrt{2}}{2} = \frac{\pi + 2\sqrt{2}}{4} = 1.493$