Worksheet 2. Answers and Commentary

This worksheet is intended to help convince students of the validity of the Second Fundamental Theorem of Calculus and to give them practice in using it. The main prerequisite is to know how to use the First Fundamental Theorem of Calculus. For Exercises 10 and 11, students also will need to know how to analyze a function using its first derivative.

1. Step one: $F(x)=\int_1^x t^3\,dt=\frac{1}{4}t^4\Big|_1^x=\frac{1}{4}x^4-\frac{1}{4}$. Step two: $F'(x)=x^3$.

2. Step one: $F(x)=\int_1^x \left(4t-t^2\right)dt=\left(2t^2-\frac{1}{3}t^3\right)\Big|_1^x=2x^2-\frac{1}{3}x^3-\left(2-\frac{1}{3}\right)$

   Step two: $F'(x)=4x-x^2$

3. Step one: $F(x)=\int_1^x \cos t\,dt=\sin t\Big|_1^x=\sin(x)-\sin(1)$

   Step two: $F'(x)=\cos x$

4. Step one: $F(x)=\int_1^{x^2} t^3\,dt=\frac{1}{4}t^4\Big|_1^{x^2}=\frac{1}{4}\left(x^2\right)^4-\frac{1}{4}(1)^4$

   Step two: $F'(x)=\left(x^2\right)^3\cdot 2x=2x^7$

5. Step one: $F(x)=\int_1^{x^2}\cos t\,dt=\sin t\Big|_1^{x^2}=\sin\left(x^2\right)-\sin(1)$

   Step two: $F'(x)=\cos\left(x^2\right)\cdot 2x$

6. Step one: $F(x)=\int_1^{x^2} 6\sqrt{t}\,dt=4t^{3/2}\Big|_1^{x^2}=4\left(x^2\right)^{3/2}-4$

   Step two: $F'(x)=6\sqrt{x^2}\cdot 2x=12x^2$

7. $F'(x)=\tan\left(x^2\right)\cdot 2x$

8. $F'(x)=\tan\left(g(x)\right)\cdot g'(x)$

9. $F'(x)=f(2x)\cdot 2$

10. (a) $H'(x) = x\cos x = 0$ when $x = \dfrac{\pi}{2}$ and $\dfrac{3\pi}{2}$.

(b) At $x = \dfrac{\pi}{2}$, $H(x)$ has a relative maximum because $H'(x)$ changes from positive to negative.

(c) At $x = \dfrac{3\pi}{2}$, $H(x)$ has a relative minimum because $H'(x)$ changes from negative to positive.

11. (a) $F(0) = \int_1^0 f(t)\,dt = -\int_0^1 f(t)\,dt = -1.3$, while $F(1) = \int_1^2 f(t)\,dt = -1.3$

(b) $F'(x) = f\left(2x\right)\cdot 2$

(c) $F'(x) = f\left(2x\right)\cdot 2 = 0$ when $x = \dfrac{1}{2}$ and $x = 2$.

(d) At $x = \dfrac{1}{2}$, $F(x)$ has a relative maximum because $F'(x) = 2f(2x)$ changes from positive to negative.

(e) At $x = 2$, $F(x)$ has a relative minimum because $F'(x) = 2f(2x)$ changes from negative to positive.