

Curriculum Module: Calculus: Functions Defined by Integrals

Let us consider the even more difficult case of $F(x) = \int_1^{x^2} f(t) dt$. Check the result for this case by performing the following two steps for each of the functions in Exercises 4–6.

Step One:

Given $f(t)$, evaluate $F(x) = \int_1^{x^2} f(t) dt$ to find $F(x)$ in terms of x .

Step Two:

Take the derivative of the result to determine $F'(x)$.

4. $f(t) = t^3$

5. $f(t) = \cos(t)$

6. $f(t) = 6\sqrt{t}$

7. If $F(x) = \int_3^{x^2} \tan(t) dt$, then $F'(x) =$ _____

8. If $F(x) = \int_3^{g(x)} \tan(t) dt$, then $F'(x) =$ _____

9. If $F(x) = \int_1^{2x} f(t) dt$, then $F'(x) =$ _____

10. Let $H(x) = \int_{\pi/2}^x t \cos(t) dt$ for $0 < x < 2\pi$.

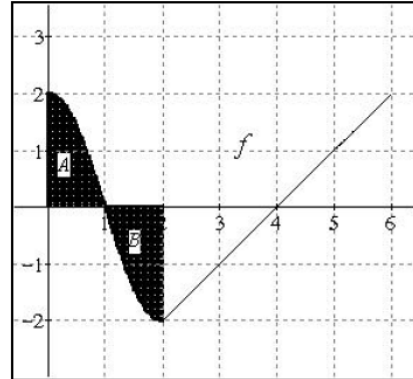
(a) Determine the critical numbers of $H(x)$.

(b) Determine which critical number corresponds to a relative maximum value of $H(x)$. Justify your answer.

(c) Determine which critical number corresponds to a relative minimum value of $H(x)$. Justify your answer.

Curriculum Module: Calculus: Functions Defined by Integrals

11. Let $F(x) = \int_1^{2x} f(t) dt$, where the graph of f on the interval $0 \leq t \leq 6$ is shown at right, and the regions A and B each have an area of 1.3.



(a) Compute $F(0)$ and $F(1)$.

(b) Determine $F'(x)$.

(c) Determine the critical numbers of $F(x)$ on the interval $0 \leq x \leq 3$.

(d) Determine which critical number of $F(x)$ corresponds to a relative maximum value of $F(x)$ on the interval $0 \leq x \leq 3$. Justify your answer.

(e) Determine which critical number of $F(x)$ corresponds to a relative minimum value of $F(x)$ on the interval $0 \leq x \leq 3$. Justify your answer.