

CALCULUS
WORKSHEET ON RIEMANN SUMS AND TRAPEZOIDAL RULE

1. A table of values for $f(t)$ is given.

t	0	20	40	60	80	100	120
$f(t)$	1.2	2.8	4.0	4.7	5.1	5.2	4.8

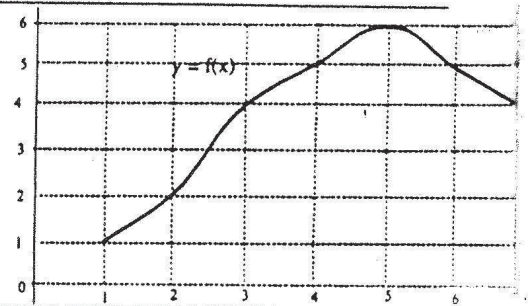
- (a) Estimate $\int_0^{120} f(t)$ by using a left Riemann sum with six subintervals.
 (b) Estimate $\int_0^{120} f(t)$ by using a right Riemann sum with six subintervals.
 (c) Estimate $\int_0^{120} f(t)$ by using a midpoint sum with three subintervals.
 (d) Estimate $\int_0^{120} f(t)$ by using the trapezoidal rule with three subintervals.

2. A table of values for $g(t)$ is given.

t	0	40	70	90	100
$g(t)$	150	180	195	184	172

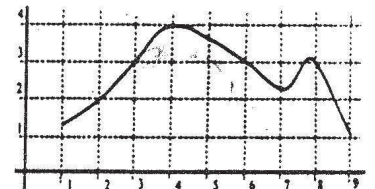
- (a) Estimate $\int_0^{100} g(t) dt$ by using a left Riemann sum with four subintervals.
 (b) Estimate $\int_0^{100} g(t) dt$ by using a right Riemann sum with four subintervals.
 (c) Estimate $\int_0^{100} g(t) dt$ by using the trapezoidal rule with four subintervals.

3. The graph of the function f over the interval $[1, 7]$ is shown. Using values from the graph, find trapezoidal rule estimates for the integral $\int_1^7 f(x) dx$ by using the indicated number of subintervals.



- (a) $n = 3$
 (b) $n = 6$

4. The graph of f over the interval $[1, 9]$ is shown in the figure. Using the data in the figure, find a midpoint approximation with 4 equal subdivisions for $\int_1^9 f(x) dx$.



5. An experiment was performed in which oxygen was produced at a continuous rate. The rate at which oxygen was produced was measured each minute and the results tabulated.

minutes	0	1	2	3	4	5	6
oxygen (cu ft/min)	0	1.4	1.8	2.2	3.0	4.2	3.6

Use the trapezoid rule to estimate the total amount of oxygen produced in 6 minutes.

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Key

1. A table of values for $f(t)$ is given.

t	0	20	40	60	80	100	120
$f(t)$	1.2	2.8	4.0	4.7	5.1	5.2	4.8

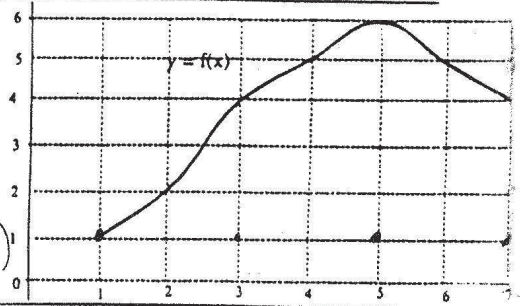
- (a) Estimate $\int_0^{120} f(t)$ by using a left Riemann sum with six subintervals. **460**
- (b) Estimate $\int_0^{120} f(t)$ by using a right Riemann sum with six subintervals. **532**
- (c) Estimate $\int_0^{120} f(t)$ by using a midpoint sum with three subintervals. **508**
- (d) Estimate $\int_0^{120} f(t)$ by using the trapezoidal rule with three subintervals. **484**

2. A table of values for $g(t)$ is given.

t	0	40	70	90	100
$g(t)$	150	180	195	184	172

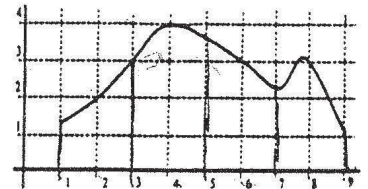
- (a) Estimate $\int_0^{100} g(t) dt$ by using a left Riemann sum with four subintervals. $L_4 = (40)(150) + 30(180) + 20(195) + 10(172) = 17,140$
- (b) Estimate $\int_0^{100} g(t) dt$ by using a right Riemann sum with four subintervals. $R_4 = 40(180) + 30(195) + 20(184) + 10(172) = 18,450$
- (c) Estimate $\int_0^{100} g(t) dt$ by using the trapezoidal rule with four subintervals. $T_4 = \frac{1}{2}(40)(150+180) + \frac{1}{2}(30)(180+195) + \frac{1}{2}(20)(195+184) + \frac{1}{2}(10)(184+172) = 17,795$

3. The graph of the function f over the interval $[1, 7]$ is shown. Using values from the graph, find trapezoidal rule estimates for the integral $\int_1^7 f(x) dx$ by using the indicated number of subintervals.



- (a) $n=3$, $\frac{6}{3 \cdot 2} = 1$, $\frac{1}{2}(1 + 2(4) + 2(6) + 4) = 25$
- (b) $n=6$, $\frac{6}{2 \cdot 6} = \frac{1}{2}$, $\frac{1}{2}(1 + 2(2) + 2(4) + 2(5) + 2(6) + 2(5) + 4) = 22$

4. The graph of f over the interval $[1, 9]$ is shown in the figure. Using the data in the figure, find a midpoint approximation with 4 equal subdivisions for



$\int_1^9 f(x) dx$. $2(2 + 4 + 3 + 3) = 24$

5. An experiment was performed in which oxygen was produced at a continuous rate. The rate at which oxygen was produced was measured each minute and the results tabulated.

minutes	0	1	2	3	4	5	6
oxygen (cu ft/min)	0	1.4	1.8	2.2	3.0	4.2	3.6

Use the trapezoid rule to estimate the total amount of oxygen produced in 6 minutes.

$\frac{6-0}{2(6)} = \frac{1}{2}(0 + 2(1.4) + 2(1.8) + 2(2.2) + 2(3) + 2(4.2) + 3.6)$

$= 14.4$