AP CALCULUS Name____________________

Optimization Problems Practice

Date________________ Period____

Solve each optimization problem.

1) A company has started selling a new type of smartphone at the price of $\$110 - 0.05x$ where x is the number of smartphones manufactured per day. The parts for each smartphone cost \$50 and the labor and overhead for running the plant cost \$6000 per day. How many smartphones should the company manufacture and sell per day to maximize profit? (Remember that Profit = Revenue - Cost)

2) A rancher wants to construct two identical rectangular corrals using 200 ft of fencing. The rancher decides to build them adjacent to each other, so they share fencing on one side. What dimensions should the rancher use to construct each corral so that together, they will enclose the largest possible area?

3) A cryptography expert is deciphering a computer code. To do this, the expert needs to minimize the product of a positive rational number and a negative rational number, given that the positive number is exactly 8 greater than the negative number. What final product is the expert looking for?

4) A rancher wants to construct two identical rectangular corrals using 400 ft of fencing. The rancher decides to build them adjacent to each other, so they share fencing on one side. What dimensions should the rancher use to construct each corral so that together, they will enclose the largest possible area?

5) Engineers are designing a box-shaped aquarium with a square bottom and an open top. The aquarium must hold 500 ft^3 of water. What dimensions should they use to create an acceptable aquarium with the least amount of glass?

6) Which point on the graph of $y = \sqrt{x}$ is closest to the point $(5, 0)$?

Answers to Optimization Problems Practice

1) p = the profit per day x = the number of items manufactured per day
Function to maximize: $p = x(110 - 0.05x) - (50x + 6000)$ where $0 \le x < \infty$
Optimal number of smartphones to manufacture per day: 600

2) A = the total area of the two corrals x = the length of the non-adjacent sides of each corral
Function to maximize: $A = 2x \cdot \dfrac{200 - 4x}{3}$ where $0 < x < 50$
Dimensions of each corall: 25 ft (non-adjecent sides) by $\dfrac{100}{3}$ ft (adjacent sides)

3) P = the product of the two numbers x = the positive number
Function to minimize: $P = x(x - 8)$ where $-\infty < x < \infty$
Smallest product of the two numbers: -16

4) A = the total area of the two corrals x = the length of the non-adjacent sides of each corral
Function to maximize: $A = 2x \cdot \dfrac{400 - 4x}{3}$ where $0 < x < 100$
Dimensions of each corall: 50 ft (non-adjecent sides) by $\dfrac{200}{3}$ ft (adjacent sides)

5) A = the area of the glass x = the length of the sides of the square bottom
Function to minimize: $A = x^2 + 4x \cdot \dfrac{500}{x^2}$ where $0 < x < \infty$
Dimensions of the aquarium: 10 ft by 10 ft by 5 ft tall

6) d = the distance from point $(5, 0)$ to a point on the curve x = the x-coordinate of a point on the curve
Function to minimize: $d = \sqrt{(x - 5)^2 + (\sqrt{x})^2}$ where $-\infty < x < \infty$
Point on the curve that is closest to the point $(5, 0)$: $\left(\dfrac{9}{2}, \dfrac{3\sqrt{2}}{2}\right)$

7) A = the area of the rectangle x = half the base of the rectangle
Function to maximize: $A = 2x\sqrt{8^2 - x^2}$ where $0 < x < 8$
Area of largest rectangle: 64

8) L = the total length of rope x = the horizontal distance from the short pole to the stake
Function to minimize: $L = \sqrt{x^2 + 4^2} + \sqrt{(15 - x)^2 + 16^2}$ where $0 \le x \le 15$
Stake should be placed: 3 ft from the short pole (or 12 ft from the long pole)

9) A = the area of the composite window x = the width of the bottom window = the diameter of the top window
Function to maximize: $A = x\left(\dfrac{18}{2} - \dfrac{x}{2} - \dfrac{\pi x}{4}\right) + \dfrac{1}{2}\pi \cdot \left(\dfrac{x}{2}\right)^2$ where $0 < x < \dfrac{72}{4 + \pi}$
Dimensions of the bottom window: $\dfrac{36}{4 + \pi}$ ft (width) by $\dfrac{18}{4 + \pi}$ ft (height)