Primary: Maximize Area   Secondary: Perimeter is given

$A = xy$ $x + 2y = 500 \quad \Rightarrow \quad x = 500 - 2y \qquad D: 0 \le y \le 250$

1.

$A(y) = 500y - 2y^2 \quad \Rightarrow \quad A'(y) = 500 - 4y \quad \Rightarrow \quad 500 - 4y = 0 \quad \Rightarrow \quad y = 125; \quad x = 250$

You must confirm whether or not a max or min.

$A''(y) = -4 \quad \Rightarrow \quad A''(125) < 0 \therefore$ it is concave down $\therefore$ a maximum.

So the dimensions are 125 ft x 250 ft

Primary: Trying to Minimize Cost   Secondary: Volume is given

$C = 10(2lw) + 6(2wh) + 6(2lh) = 60w^2 + 48wh$ $V = 50 = lwh = 3w^2h \Rightarrow h = \dfrac{50}{3w^2} \Rightarrow D: w > 0$

$C(w) = 60w^2 + \dfrac{800}{w}$

2. $C'(w) = 120w - 800w^{-2} = \dfrac{120w^3 - 800}{w^2} \quad \Rightarrow \quad 120w^3 - 800 = 0 \quad \Rightarrow \quad w = \sqrt[3]{\dfrac{800}{120}} = 1.8821$

You must confirm whether or not a max or min.

$C''(w) = 120 + 1600w^{-3} \quad \Rightarrow \quad C''(1.8821) > 0 \therefore$ it is concave up $\therefore$ a minimum.

$C(1.8821) = 6w^2 + \dfrac{800}{w} = \$637.60$

Primary: Maximize Volume   Secondary: Material for Surface Area is given

$V = lwh = w^2h$ $10 = 2lw + 2wh + 2lh = 2w^2 + 4wh \Rightarrow h = \dfrac{5 - w^2}{2w} \Rightarrow D: 0 < w < \sqrt{5}$

$V(w) = w^2\left(\dfrac{5 - w^2}{2w}\right) = \dfrac{1}{2}\left(5w - w^3\right)$

3. $V'(w) = \dfrac{1}{2}\left(5 - 3w^2\right) \quad \Rightarrow \quad 5 - 3w^2 = 0 \quad \Rightarrow \quad w = \pm\sqrt{\dfrac{5}{3}} = \pm 1.2910$

You must confirm whether or not a max or min.

$V''(w) = -3w \quad \Rightarrow \quad V''(1.2910) < 0 \therefore$ it is concave down $\therefore$ a maximum.

$l = w = 1.2910 \qquad h = 1.2910 \qquad Max\ \ Volume: 1.2910\,ft \text{ x } 1.2910\,ft \text{ x } 1.2910\,ft$

4.

Primary: Minimize Area Secondary: Volume is given

$A = 2\pi rh + 2\pi r^2$ $\quad\quad V = 1500 = \pi r^2 h \Rightarrow h = \frac{1500}{\pi r^2} \Rightarrow D: r > 0$

$$A(r) = 2\pi r\left(\frac{1500}{\pi r^2}\right) + 2\pi r^2 = 2\pi r^2 + \frac{3000}{r} \Rightarrow$$

$$A'(r) = 4\pi r - \frac{3000}{r^2} = \frac{4\pi r^3 - 3000}{r^2} \Rightarrow 4\pi r^3 - 3000 = 0 \Rightarrow r = \sqrt[3]{\frac{750}{\pi}} = 6.2035$$

You must confirm whether or not a max or min.

$A'(6) < 0$ and $A'(7) > 0$ By the 1st derivative test 6.2035 is a min

So the dimensions are $r = 6.2035\,cm$ and $h = 12.4070\,cm$

5.

Primary: Minimize Area No Secondary since primary is in terms of only one variable, *h*.

$V(h) = h(14 - 2h)(10 - 2h) = 140h - 48h^2 + 4h^3 \quad D: 0 \le h \le 5$

$$V'(h) = 140 - 96h + 12h^2 \Rightarrow 140 - 96h + 12h^2 = 0 \Rightarrow h = \frac{12 \pm \sqrt{39}}{3} = 1.9183,\ 6.0817$$

You must confirm whether or not a max or min.

$V''(h) = -96 + 24h \Rightarrow V''(1.9183) < 0 \therefore$ it is concave down $\therefore$ a maximum.

$V(0) = 0 \quad V(1.9183) = 120.1644 \quad V(5) = 0 \therefore h = 1.9183\,in$

6.

Primary: Maximize print area Secondary: Total Area is given

$A = (w - 2)(h - 3.5)$ $\quad\quad 200 = wh \Rightarrow h = \frac{200}{w} \Rightarrow D: w > 0$

$$A = (w - 2)\left(\frac{200}{w} - 3.5\right) = 207 - 3.5w - \frac{400}{w}$$

$$A'(w) = -3.5 + \frac{400}{w^2} = \frac{400 - 3.5w^2}{w^2} \Rightarrow 400 - 3.5w^2 = 0 \Rightarrow w = \pm\sqrt{\frac{400}{3.5}} = \pm 10.6904$$

You must confirm whether or not a max or min.

$A''(w) = -\frac{800}{w^3} \Rightarrow A''(10.6904) < 0 \therefore$ it is concave down $\therefore$ a maximum.

Dimensions for Max print : $18.7084\ in \times 10.6904\ in$