



# Quadratic Equations

## I. Quadratic Equations

a. Definition: A quadratic equation with one unknown variable is an equation in which there appears an exponent of 2 on the unknown (and sometimes an exponent of 1 as well).

- For instance:  $x^2 + 4 = 0$  is quadratic
- $x^2 + 2x = 0$  is quadratic
- $x^2 + 2x + 1 = 0$  is quadratic
- $x^2 + 1 = 4x^2 - 2x$  is quadratic

b. The *General Form* of a quadratic equation is:  $ax^2 + bx + c = 0$ .

- $a$  = coefficient of  $x^2$  term
- $b$  = coefficient of  $x$  term
- $c$  = constant term (a number)

## II. Methods of Solving Quadratic Equations:

- a. Put equation in standard form. This may involve removing parentheses, combining like terms, and moving all terms to one side of the equation.
- b. If the left-hand side factors, set each factor equal to zero and solve the 2 linear equations. Then check your answers!!

Ex)  $x^2 + 3x - 4 = 0$   
 $(x + 4)(x - 1) = 0$   
 $x + 4 = 0$                        $x - 1 = 0$   
 $x = -4$                       or                       $x = 1$   
Answer:  $x = -4, x = 1$

Ex)  $(x - 2)^2 = 4$   
 $x^2 - 4x + 4 = 4$   
 $x^2 - 4x = 0$  → Equation is now in standard form.  
 $x(x - 4) = 0$   
 $x = 0$      $x = 4$   
Answer:  $x = 0, x = 4$

- c. If the left-hand side does not factor, use the quadratic formula to solve the equation. Then check your answers!!

**Quadratic formula:**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Ex)  $x^2 + 3x + 1 = 0$  (This will not factor!)  
 $a = 1$     $b = 3$     $c = 1$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{9 - 4}}{2}$$

$$x = \frac{-3 \pm \sqrt{5}}{2}$$

There are two answers for  $x$ :  $x = \frac{-3 + \sqrt{5}}{2}$  or  $x = \frac{-3 - \sqrt{5}}{2}$

**NOTE:** If the number which appears under the radical in the quadratic formula is negative there is no solution for  $x$ , since it is impossible to take the square root of a negative number.

### III. More Examples:

1.  $x^2 - 3x = 0$  ←———— This factors.  
 $x(x-3) = 0$   
 $x = 0$  or  $x - 3 = 0$   
 $x = 3$   
Answer:  $x = 0, x = 3$

2.  $x^2 - 9 = 0$  ←———— This factors.  
 $(x+3)(x-3) = 0$   
 $x+3 = 0$  or  $x-3 = 0$   
 $x = -3$   $x = 3$   
Answer:  $x = -3, x = 3$

3.  $x^2 - 12x + 36 = 0$  ←———— This factors.  
 $(x-6)(x-6) = 0$   
 $x = 6$  or  $x = 6$  (both answers are the same)  
Answer:  $x = 6$

4.  $3x^2 - x - 2 = 0$  ←————  
Method 1: Factoring  
 $(3x+2)(x-1) = 0$   
 $3x = -2$   $x = 1$   
 $x = -\frac{2}{3}$

This factors and may be solved by factoring but you can solve any quadratic by the quadratic formula. It will be illustrated with this example that you will obtain the same answers from either the quadratic formula or by factoring.

#### Method 2: Quadratic Formula

$$3x^2 - x - 2 = 0$$
$$a = 3 \quad b = -1 \quad c = -2$$
$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(-2)}}{2(3)}$$

$$x = \frac{1 \pm \sqrt{1+24}}{6} = \frac{1 \pm \sqrt{25}}{6}$$

$$x = \frac{1+5}{6} \quad \text{or} \quad x = \frac{1-5}{6}$$

$$x = 1 \quad \text{or} \quad x = \frac{-4}{6} = \frac{-2}{3}$$

Practice Problems:

Solve the following quadratic equations by factoring.

1.  $x^2 - 2x = 0$

2.  $p^2 - 9 = 0$

3.  $x^2 + 6x + 8 = 0$

4.  $T^2 - 6T - 16 = 0$

5.  $q^2 - 5q = -6$

6.  $4y^2 = 25$

7.  $9d^2 + 64 = 48d$

8.  $3e^2 + 3e - 36 = 0$

9.  $30h^2 + 23h = -4$

10.  $(5a - 1)^2 = 9$

11.  $(5s - 15)^2 = 64$

Solve the following quadratic equations by the quadratic formula.

12.  $x^2 - 6x - 4 = 0$

13.  $x^2 + 8x + 4 = 0$

14.  $2x^2 + 3x - 1 = 0$

15.  $-2d^2 + 3d - 5 = 0$

16.  $3u^2 = 10u - 3$

17.  $5b^2 + 3 = -9b$

18.  $2w(w+1) = 7$

19.  $3r(r-1) + 2(3r+4) = 18$

20.  $e + \frac{3}{5} = 3 + \frac{1}{e}$

21.  $2s^2 - .4s = 2.2$

22.  $\frac{x+3}{x-1} = \frac{x-1}{x-2} - 2$

23.  $\frac{x+2}{x+1} - 4 = \frac{x+1}{x+3}$

24.  $t + 2 = \frac{t^2}{3}$

Solve the following equations by any appropriate method.

25.  $x^2 - 2x + 3 = 0$

26.  $3x^2 + 5x - 5 = 1 - 2x$

27.  $x^2 = 10x$

28.  $\frac{2}{x} + 1 = x - \frac{5}{2}$

29.  $(7p - 2)^2 = 18$

30.  $3x^2 - 2x - 3 = 0$

31.  $(T^2 - 5)^2 = 9$

32.  $\frac{x}{2} = \frac{1}{5} - \frac{1}{x}$

33.  $3x + 2(x - 5) = 7 - (x + 3)^2$

Answers to Quadratic Equations:

1.  $x=0, x=2$
2.  $p=3, p=-3$
3.  $x=-4, x=-2$
4.  $T=8, T=-2$
5.  $q=3, q=2$
6.  $y=\frac{5}{2}, y=-\frac{5}{2}$
7.  $d=\frac{8}{3}$
8.  $e=-4, e=3$
9.  $h=\frac{-4}{15}, h=\frac{-1}{2}$
10.  $a=\frac{4}{5}, a=-\frac{2}{5}$
11.  $s=\frac{23}{5}, s=\frac{7}{5}$
12.  $x=3\pm\sqrt{13}$
13.  $x=-4\pm 2\sqrt{3}$
14.  $x=\frac{-3\pm\sqrt{17}}{4}$
15. no real roots
16.  $u=\frac{1}{3}, u=3$
17.  $b=\frac{-9\pm\sqrt{21}}{10}$
18.  $w=\frac{-1\pm\sqrt{15}}{2}$
19.  $r=\frac{-3\pm\sqrt{129}}{6}$
20.  $e=\frac{6\pm\sqrt{61}}{5}$
21.  $s=\frac{1\pm\sqrt{111}}{10}$
22.  $x=\frac{3\pm\sqrt{33}}{4}$
23.  $x=\frac{-13\pm\sqrt{57}}{8}$
24.  $t=\frac{3\pm\sqrt{33}}{2}$
25. no real roots
26.  $x=\frac{2}{3}, x=-3$
27.  $x=0, x=10$
28.  $x=4, x=\frac{-1}{2}$
29.  $x=\frac{2\pm 3\sqrt{2}}{7}$
30.  $x=\frac{1\pm\sqrt{10}}{3}$
31.  $t=\pm 2\sqrt{2}, t=\pm\sqrt{2}$
32. no real roots
33.  $x=\frac{-11\pm\sqrt{153}}{2}$