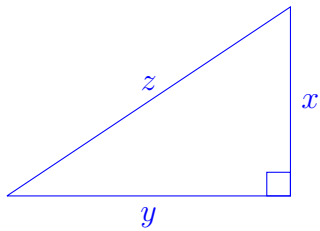# Related Rates Word Problems
SOLUTIONS

(1) One car leaves a given point and travels north at 30 mph. Another car leaves 1 HOUR LATER, and travels west at 40 mph. At what rate is the distance between the cars changing at the instant the second car has been traveling for 1 hour?

Set up the problem by extracting information in terms of the variables x, y, and z, as pictured on the triangle:
- First sentence: $\frac{dx}{dt} = 30$ and $x(t) = 30t$.
- Second: $\frac{dy}{dt} = 40$ and $y(t) = 40(t-1)$ (Start one hour late!)
- Goal: Find $\frac{dz}{dt}$ at $t = 2$.

The property that combines the sides of a triangle is Pythagorean Theorem:

$$x^2 + y^2 = z^2.$$

At $t = 2$, $x(2) = 60$ and $y(2) = 40$. Using Pythagorean Theorem: $z(2) = \sqrt{60^2 + 40^2} \approx 72.111$. Taking the derivative in t:
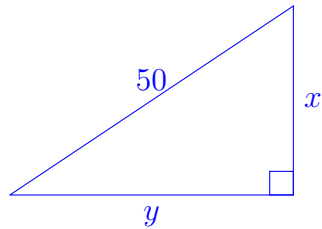
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}.$$

Plug in:

$$2 \cdot 60 \cdot 30 + 2 \cdot 40 \cdot 40 = 2 \cdot 72.111 \cdot \frac{dz}{dt}.$$

Thus, $\frac{dz}{dt} \approx 47.150$.

(2) A 50ft ladder is placed against a large building. The base of the ladder is resting on an oil spill, and it slips at the rate of 3 ft. per minute. Find the rate of change of the height of the top of the ladder above the ground at the instant when the base of the ladder is 30 ft. from the base of the building.

Organizing information:

- $\dfrac{dy}{dt} = 3$

- Goal: Find $\dfrac{dx}{dt}$ when $y = 30$.

We use Pythagorean Theorem again:

$$x^2 + 30^2 = 50^2 \implies x = 40.$$

And differentiating (notice how the hypotenuse is constant):

$$2xx' + 2yy' = 0x' \qquad = \dfrac{-2yy'}{2x} = \dfrac{-yy'}{x}$$

Plugging in, $x' = -30 \cdot 3 \div 40 = -2.25$.

Note: x' is negative, that means the distance x is decreasing—the ladder is slipping down the building.

(3) A stone dropped in a pond sends out a circular ripple whose radius increases at a constant rate of 4 ft/sec. After 12 seconds, how rapidly is the area inclosed by the ripple increasing?

Organizing information:

- $\dfrac{dr}{dt} = 4$

- Goal: Find $\dfrac{dA}{dt}$ when $t = 12$.

We use the area formula for a circle.

$$A = \pi r^2$$

Differentiate both sides with respect to t:

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

Plug in $\frac{dr}{dt} = 4$. When t = 12 seconds, r = 4*12 = 48.

$$\frac{dA}{dt} = 2\pi(48) * 4 = 384\pi \, ft^2/sec$$

(4) A spherical balloon is being inflated so that its diameter is increasing at a rate of 2 cm/min. How quickly is the volume of the balloon increasing when the diameter is 10 cm?

Organizing information:

- $\dfrac{dd}{dt} = 2$

- Goal: Find $\dfrac{dV}{dt}$ when $d = 10$.

We use the volume formula for a sphere, but rewrite it with the diameter

$$V = \frac{4}{3}\pi r^3$$
$$V = \frac{4}{3}\pi(\frac{d}{2})^3$$
$$V = \frac{\pi}{6}d^3$$

Differentiate both sides with respect to t:

$$\frac{dV}{dt} = 3\frac{\pi}{6}d^2\frac{dd}{dt}$$

Plug in $\frac{dd}{dt} = 2$ and $d = 10$

$$\frac{dV}{dt} = 3\frac{\pi}{6}(10)^2 * 2 = 100\pi cm^3/min$$

(5) The radius of a cylinder is **increasing** at a rate of 1 meter per hour, and the height of the clinder is **decreasing** at a rate of 4 meters per hour. At a certain instant, the base radius is 5 meters and the height is 8 meters. What is the rate of change of the volume of the cylinder at the instant?

Organizing information:

- $\dfrac{dr}{dt} = 1$, $\dfrac{dh}{dt} = -4$
- Goal: Find $\dfrac{dV}{dt}$ when $r = 5$, $h = 8$.

We use the volume formula for a cylinder

$$V = \pi r^2 h$$

Differentiate both sides with respect to t: (you have a product rule on the right side)

$$\frac{dV}{dt} = (\pi r^2) * \left(\frac{dh}{dt}\right) + h\left(2\pi r \frac{dr}{dt}\right)$$

Plug in $\frac{dr}{dt} = 1$, $\frac{dh}{dt} = -4$, $r = 5$ and $h = 8$.

$\dfrac{dV}{dt} = (\pi(5)^2) * (-4) + 8 * (2\pi(5)(1)) = -200\pi + 80\pi = -120m^3/hour$

(6) A person who is 6 feet tall is walking away from a lamp post at the rate of 40 feet per minute. When the person is 10 feet from the lamp post, his shadow is 20 feet long. Find the rate at which the length of the shadow is increasing when he is 30 feet from the lamp post.

The diagram and labeling is similar to a problem done in class.
Organizing information:

- $\dfrac{dx}{dt} = 40$, when x = 10, s=20

- Goal: Find $\dfrac{ds}{dt}$ when $x = 30$.

We set up a ratio of similar triangles.

$$\frac{x+s}{h} = \frac{s}{6}$$

The height of the pole is a constant. We solve for h by using that when $x = 10$, $s = 20$.

$$\frac{10+20}{h} = \frac{20}{6}$$
$$6(30) = 20h$$
$$h = 180/20 = 9$$

Now rewrite our orginal ratio equation with the constant height solved for:

$$\frac{x+s}{9} = \frac{s}{6}$$
$$6x + 6s = 9s$$
$$6x = 3s$$

Differentiate both sides with respect to t and solve for $\frac{ds}{dt}$

$$6\frac{dx}{dt} = 3\frac{ds}{dt}$$

Plug in $\frac{dx}{dt} = 40$, and solve for $\frac{ds}{dt}$

$$\frac{ds}{dt} = 80 ft/min$$