Calculus 1
WKST/Key- Implicit Differentiation

Name:________________________________

Date:________________________________

On problems 1 – 3, find $\frac{dy}{dx}$.

1. $x^3 + xy + y^3 = 1$

2. $y - x\sin y = 3$

3. $x + \tan(xy) = 0$

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4. If $y = xy + x^2 + 1$, find $\frac{dy}{dx}$ when $x = -1$.

5. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$, given $y^2 + 2y = 2x + 1$.

6. If $x^3 + y^3 = 8$, show that the second derivative of y with respect to x is $-\frac{16x}{y^5}$.

7. Consider the curve defined by the equation $y + \cos y = x + 1$ for $0 \le y \le 2\pi$.

(a) Find $\frac{dy}{dx}$ in terms of y.

(b) Write an equation for each vertical tangent to the curve.

(c) Find $\frac{d^2y}{dx^2}$ in terms of y.

8. Consider the curve $y^2 = 4 + x$ and the chord AB joining points $\mathrm{A}(-4, 0)$ and B(0, 2) on the curve. Find the x- and y-coordinates of the point on the curve where the tangent line is parallel to chord AB.

9. The function f is differentiable for all real numbers. The point $\left(3, \frac{1}{4}\right)$ is on the graph of $y = f(x)$, and the slope at each point (x, y) on the graph is given by $\frac{dy}{dx} = y^2(6 - 2x)$.

Find $\frac{d^2y}{dx^2}$, and evaluate it at the point $\left(3, \frac{1}{4}\right)$.

10. Consider the curve given by $xy^2 - x^3y = 6$.

(a) Find $\frac{dy}{dx}$.

(b) Find all points on the curve whose x-coordinate is 1, and write an equation for the tangent line to the curve at each of these points.

(c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

Answers to Worksheet on Implicit Differentiation

1. $\dfrac{-3x^2 - y}{x + 3y^2}$

2. $\dfrac{\sin y}{1 - x\cos y}$

3. $\dfrac{-1 - y\sec^2(xy)}{x\sec^2(xy)}$ or $\dfrac{-\cos^2(xy) - y}{x}$

4. $\dfrac{y + 2x}{1 - x}, -\dfrac{1}{2}$

5. $\dfrac{1}{y+1}, \dfrac{-1}{(y+1)^3}$

6. $-\dfrac{x^2}{y^2}, -\dfrac{16x}{y^5}$

7. (a) $\dfrac{1}{1 - \sin y}, y \neq \dfrac{\pi}{2}$

(b) $x = \dfrac{\pi}{2} - 1$

(c) $\dfrac{\cos y}{(1 - \sin y)^3}$

8. (- 3, 1)

9. $-\dfrac{1}{8}$

10. (a) $\dfrac{3x^2 y - y^2}{2xy - x^3}$

(b) (1, 3), $y = 3$; $(1, -2)$, $y + 2 = 2(x - 1)$

(c) $x = 0$ but no such point on the curve;

$x = -\sqrt[10]{576}$ $y = \sqrt[5]{18}$