Worksheet on Chain Rule

What you don't finish is your homework for the night.

Show all work, including rewriting the original problem in a more useful way. No calculator unless otherwise stated.

1. Find the derivative of the following functions with respect to the independent variable.

(a) $y=(2x-7)^3$

$y' = 3(2x-7)^2 \cdot 2$

$= \boxed{6(2x-7)^2}$

(b) $y=\dfrac{1}{t^2+3t-1}$

$y=(t^2+3t-1)^{-1}$

$y' = -1(t^2+3t-1)^{-2} \cdot (2t+3)$

$\boxed{y' = \dfrac{-(2t+3)}{(t^2+3t-1)^2}}$

(c) $y=\left(\dfrac{1}{t-3}\right)^2$

$y = ((t-3)^{-1})^2$

$= (t-3)^{-2}$

$y' = -2(t-3)^{-3}$

$\boxed{= \dfrac{-2}{(t-3)^3}}$

(d) $y=\csc^3\left(\dfrac{3x}{2}\right)$

$= (\csc(\tfrac{3x}{2}))^3$

$y' = 3(\csc(\tfrac{3x}{2}))^2 \cdot -\csc(\tfrac{3x}{2})\cot(\tfrac{3x}{2}) \cdot \tfrac{3}{2}$

$= \boxed{-\tfrac{9}{2}(\csc(\tfrac{3x}{2}))^2 \csc(\tfrac{3x}{2})\cot(\tfrac{3x}{2})}$

or

$\boxed{-\tfrac{9}{2}(\csc(\tfrac{3x}{2}))^3 \cot(\tfrac{3x}{2})}$

(e) $y=3\sec^2(\pi t-1)$

$y = 3(\sec(\pi t-1))^2$

$y' = 6(\sec(\pi t-1))' \sec(\pi t-1)\tan(\pi t-1) \cdot \pi$

$= \boxed{6\pi (\sec(\pi t-1))^2 \tan(\pi t-1)}$

or

$\boxed{6\pi \sec^2(\pi t-1)\tan(\pi t-1)}$

(f) $y=\sin\sqrt[3]{x}+\sqrt[3]{\sin x}$

$y = \sin x^{1/3} + (\sin x)^{1/3}$

$y' = \cos x^{1/3} \cdot \tfrac{1}{3}x^{-2/3} + \tfrac{1}{3}(\sin x)^{-2/3} \cdot \cos x$

$\boxed{y' = \dfrac{\cos\sqrt[3]{x}}{3x^{2/3}} + \dfrac{\cos x}{3\sqrt[3]{\sin^2 x}}}$

(g) $y=x^2\tan\dfrac{1}{x}$

$y = x^2 \tan x^{-1}$

$y' = x^2 \sec^2(x^{-1}) \cdot -1x^{-2} + \tan(x^{-1}) \cdot 2x$

$y' = \dfrac{-x^2}{x^2}\sec^2(\tfrac{1}{x}) + \tan(\tfrac{1}{x}) \cdot 2x$

$\boxed{y = -\sec^2(\tfrac{1}{x}) + 2x\tan(\tfrac{1}{x})}$

(h) $r=\sec 2\theta \tan 2\theta$

No Room

☹

2. Find the equation of the tangent line for each of the following at the indicated point.

(a) $s(t)=\sqrt{t^2+2t+8}$ at $t=2$ $y=4$

$s'(t) = \tfrac{1}{2}(t^2+2t+8)^{-1/2}(2t+2)$

$s'(2) = \tfrac{1}{8} \cdot 10 = \tfrac{5}{4}$

$\boxed{y-4 = \tfrac{5}{4}(x-2)}$

(b) $f(t)=\dfrac{3t+2}{t-1}$ at $(0,-2)$

$f'(t) = \dfrac{(t-1)\cdot 3 - (3t+2)\cdot 1}{(t-1)^2}$

$f'(0) = \dfrac{-3-2}{1} = -5$

$\boxed{y+2 = -5x}$

4. Find the second derivative of each of the following functions. Remember to simplify early and often.

(a) $f(x) = 2(x^2-1)^3$

$f'(x) = 12x(x^2-1)^2$

$f''(x) = 12x \cdot 2(x^2-1)^1 \cdot 2x + (x^2-1)^2 \cdot 12$

$= 12(x^2-1)(4x^2 + x^2 - 1) = \boxed{12(x^2-1)(5x^2-1)}$

(b) $f(x) = \sin(x^2)$

$f'(x) = 2x\cos(x^2)$

$f''(x) = 2x(-\sin(x^2)) \cdot 2x + \cos(x^2) \cdot 2$

$= -2(4x^2\sin(x^2) - \cos(x^2))$

or

$\boxed{2(-4x^2\sin(x^2) + \cos(x^2))}$

5. If $h(x) = \tan(2x)$, evaluate $h''(x)$ at $\left(\frac{\pi}{6}, \sqrt{3}\right)$. Simplify early and often.

$h'(x) = 2\sec^2(2x) = 2(\sec(2x))^2$

$h''(x) = 4(\sec(2x))^1 \cdot \sec(2x)\tan(2x) \cdot 2$

$= 8\sec^2(2x)\tan(2x)$

$h''(\pi/6) = 8\left(\sec\frac{\pi}{3}\right)^2 \tan\left(\frac{\pi}{3}\right) = 8 \cdot (2)^2\sqrt{3} = \boxed{32\sqrt{3}}$

6. If $g(5) = -3$, $g'(5) = 6$, $h(5) = 3$, and $h'(5) = -2$, find $f'(5)$ (if possible) for each of the following. If it is not possible, state what additional information is required.

(a) $f(x) = \dfrac{g(x)}{h(x)}$

$f'(x) = \dfrac{h(x)g'(x) - g(x)(h'(x))}{(h(x))^2}$

$f'(5) = \dfrac{3 \cdot 6 - (-3)(-2)}{3^2}$

$= \dfrac{18-6}{9} = \dfrac{4}{3}$

(b) $f(x) = g(h(x))$

$f'(x) = g'(h(x)) \cdot h'(x)$

$f'(5) = g'(h(5)) \cdot h'(5)$

$= g'(3) \cdot (-2)$

Not possible

Need $g'(3)$

(c) $f(x) = g(x)h(x)$

$f'(x) = g(x)h'(x) + h(x)g'(x)$

$f'(5) = -3 \cdot (-2) + 3(6)$

$f'(5) = 24$

(d) $f(x) = [g(x)]^3$

$f'(x) = 3(g(x))^2 \cdot g'(x)$

$f'(5) = 3(-3)^2 \cdot 6$

$= 162$

(e) $f(x) = g(x + h(x))$

$f'(x) = g'(x + h(x)) \cdot (1 + h'(x))$

$f'(5) = g'(5+3)(1 + (-2))$

$f'(5) = g'(8)(-1)$

Not possible

Need $g'(8)$

(f) $f(x) = (g(x) + h(x))^{-2}$

$f'(x) = \dfrac{-2(g'(x) + h'(x))}{(g(x) + h(x))^3}$

$f'(5) = \dfrac{-2(6 + -2)}{(-3+3)^3}$

$f'(5)$ is undefined (vertical tangent!)

Multiple Choice

_____ 14. If $f(x)=\dfrac{1}{\sqrt{x^2+3}}$, find $f'(x)$.

(A) $f'(x)=-\dfrac{x}{\sqrt{\left(x^2+3\right)^3}}$

(B) $f'(x)=\dfrac{x}{\sqrt{x^2+3}}$

(C) $f'(x)=-\dfrac{x}{\left(x^2+3\right)\sqrt{2x}}$

(D) $f'(x)=-\dfrac{1}{2\sqrt{\left(x^2+3\right)^3}}$

(E) $f'(x)=-\dfrac{x^2+3x}{x^2+3}$

$f(x)=(x^2+3)^{-1/2}$

$f'(x)= \dfrac{-1}{2}(x^2+3)^{-3/2}\cdot 2x$

$= \dfrac{-x}{(x^2+2)^{3/2}}$

15. If $g(x)=(1-x)^3(4x+1)$, then $g'(x)=$

(A) $-12(1-x)^2$

(B) $(1-x)^2(1+8x)$

(C) $(1-x)^2(1-16x)$

(D) $3(1-x)^2(4x+1)$

(E) $(1-x)^2(16x+7)$

15. $g'(x)=(1-x)^3\cdot 4 + (4x+1)\,3(1-x)^2\cdot(-1)$

$=(1-x)^2\left(4(1-x) + -3(4x+1)\right)$

$=(1-x)^2(4-4x-12x-3)$

$=(1-x)^2(1-16x)$

16. $\dfrac{d}{dx}\left[\left(\dfrac{x^2-3}{5x^2-9}\right)^5\right]=$

(A) $\dfrac{10x(x^2-3)^4(10x^2-17)}{(5x^2-9)^6}$

(B) $\dfrac{-10x(x^2-3)^4(5x^2-16)}{(5x^2-9)^5}$

(C) $\dfrac{-240x(x^2-3)^4}{(5x^2-9)^6}$

(D) $\dfrac{60x(x^2-3)^4}{(5x^2-9)^6}$

(E) $\dfrac{100x(x^2-3)^4}{(5x^2-9)^6}$

16. $5\left(\dfrac{x^2-3}{5x^2-9}\right)^4 \cdot \ldots$

Must be able to do on your own :)