

Differentiation - Trigonometric Functions

Date _____ Period _____

Differentiate each function with respect to x .

1) $f(x) = \sin 2x^3$

$$\begin{aligned} f'(x) &= \cos 2x^3 \cdot 6x^2 \\ &= 6x^2 \cos 2x^3 \end{aligned}$$

2) $y = \tan 5x^3$

$$\begin{aligned} \frac{dy}{dx} &= \sec^2 5x^3 \cdot 15x^2 \\ &= 15x^2 \cdot \sec^2 5x^3 \end{aligned}$$

3) $y = \sec 4x^5$

$$\begin{aligned} \frac{dy}{dx} &= \sec 4x^5 \cdot \tan 4x^5 \cdot 20x^4 \\ &= 20x^4 \sec 4x^5 \cdot \tan 4x^5 \end{aligned}$$

4) $y = \csc 5x^5$

$$\begin{aligned} \frac{dy}{dx} &= -\csc 5x^5 \cdot \cot 5x^5 \cdot 25x^4 \\ &= -25x^4 \csc 5x^5 \cdot \cot 5x^5 \end{aligned}$$

5) $y = (2x^5 + 3)\cos x^2$

$$\begin{aligned} \frac{dy}{dx} &= (2x^5 + 3) \cdot -1\sin x^2 \cdot 2x + \cos x^2 \cdot 10x^4 \\ &= 2x(-2x^5 \sin x^2 - 3\sin x^2 + 5x^3 \cos x^2) \end{aligned}$$

6) $y = \frac{-2x^2 - 5}{\cos 2x^3}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{\cos 2x^3 \cdot -4x - (-2x^2 - 5) \cdot -1\sin 2x^3 \cdot 6x^2}{\cos^2 2x^3} \\ &= \frac{2x(-2\cos 2x^3 - 6x^3 \sin 2x^3 - 15x \sin 2x^3)}{\cos^2 2x^3} \end{aligned}$$

7) $f(x) = \sin^3 x^5$

$$\begin{aligned} f'(x) &= 3 \cdot \sin^2 x^5 \cos x^5 \cdot 5x^4 \\ &= 15x^4 \cdot \sin^2 x^5 \cos x^5 \end{aligned}$$

8) $f(x) = \cos(-3x^2 + 2)^2$

$$\begin{aligned} f'(x) &= -\sin(-3x^2 + 2)^2 \cdot 2(-3x^2 + 2) \cdot -6x \\ &= 12x \sin(-3x^2 + 2)^2 \cdot (-3x^2 + 2) \end{aligned}$$

9) $y = \sin\left(\frac{x}{2}\right)^3$

$$\begin{aligned} y' &= \cos\left(\frac{x}{2}\right)^3 \cdot 3\left(\frac{x}{2}\right)^2 \cdot \frac{1}{2} \\ &= \frac{3}{2}\left(\frac{x}{2}\right)^2 \cos\left(\frac{x}{2}\right)^3 \end{aligned}$$

10) $f(x) = \cot(2x^3 + 1)^2$

$$\begin{aligned} f'(x) &= -\csc^2(2x^3 + 1)^2 \cdot 2(2x^3 + 1)^1 \cdot 6x^2 \\ &= -12x^2(2x^3 + 1)\csc^2(2x^3 + 1)^2 \end{aligned}$$

$$11) y = \sin^2(3x)^4$$

$$\begin{aligned} y' &= 2 \sin(3x)^4 \cdot \cos(3x)^4 \cdot 4(3x)^3 \cdot 3 \\ &= 24(3x)^3 \sin(3x)^4 \cos(3x)^4 \end{aligned}$$

$$12) f(x) = \csc^3(2x+1)^2$$

$$\begin{aligned} f'(x) &= 3 \csc^2(2x+1)^2 \cdot -\csc(2x+1)^2 \cot(2x+1)^2 \cdot 2(2x+1)^1 \cdot 2 \\ &= -12(2x+1) \csc^2(2x+1)^2 \csc(2x+1)^2 \cot(2x+1)^2 \\ &= -12(2x+1) \csc^3(2x+1)^2 \cot(2x+1)^2 \end{aligned}$$

$$13) \text{ Find the 2nd derivative of } f(x) = \sec(2x-1)$$

$$\begin{aligned} f'(x) &= \sec(2x-1) \tan(2x-1) \cdot 2 \\ &= 2 \sec(2x-1) \tan(2x-1) \end{aligned}$$

$$\begin{aligned} f''(x) &= 2 \sec(2x-1) \sec^2(2x-1) \cdot 2 + \tan(2x-1) \cdot 2 \sec(2x-1) \tan(2x-1) \cdot 2 \\ &= 4 \sec^3(2x-1) + 4 \tan^2(2x-1) \cdot \sec(2x-1) \end{aligned}$$

Or

$$= 4 \sec(2x-1) \left[\sec^2(2x-1) + \tan^2(2x-1) \right]$$