

Calculus-Related Factoring

In calculus, you will need to factor and simplify expressions resulting from finding derivatives of functions using the product and quotient rules. Use the examples to guide you in factoring the rest of the problems.

Ex: Factor completely to simplify

$$\begin{aligned}(2x+1)(6x) + (3x^2-2)(2) \\ &= 2[(2x+1)(3x) + (3x^2-2)] \\ &= 2[6x^2 + 3x + 3x^2 - 2] \\ &= 2[9x^2 + 3x - 2] \\ &= 2(3x+2)(3x-1)\end{aligned}$$

Ex: Factor completely to simplify:

$$\begin{aligned}3(x-1)^2(x+2)^4 + 4(x-1)^3(x+2)^3 \\ &= (x-1)^2(x+2)^3[3(x+2) + 4(x-1)] \\ &= (x-1)^2(x+2)^3[3x+6+4x-4] \\ &= (x-1)^2(x+2)^3(7x+2)\end{aligned}$$

Exercises. Factor completely and simplify.

1. $2(x-1)^2 + 5(x-1)$

3. $3x^2(4x-12)^2 + x^3(2)(4x-12)(4)$

5. $3(x-1)^2(x+2)^4 + 4(x-1)^3(x+2)^3$

7. $6(x^2+3x+1)(2x+3)^2 + 6(x^2+3x+1)^2$

9. $\frac{2x(x+6)^4 - x^2(4)(x+6)^3}{(x+6)^8}$

11. $\frac{(x-1)^2(1-2x) - 2(2+x-x^2)(x-1)}{(x-1)^4}$

13. $\frac{3(x+2)^2(x-3)^2 - (x+2)^3(2)(x-3)}{(x-3)^4}$

2. $9x^2(x^4+2) + 3x^3(4x^3)$

4. $5(x^2+4)^4(2x)(x-2)^4 + (x^2+4)^5(4)(x-2)^3$

6. $2(x+1)(x-1)^{-2} - 2(x+1)^2(x-1)^{-3}$

8. $60x^3(1-3x^2)^5(5x^4-1)^2 - 30x(5x^4-1)^3(1-3x^2)^4$

10. $\frac{(x-1)^3 - 3(x-5)(x-1)^2}{(x-1)^6}$

12. $\frac{8(1-3x)^2(2x-1)^3 + 6(2x-1)^4(1-3x)}{(1-3x)^4}$

14. $\frac{3(x+2)^2 - 6x(x+2)}{x+2}$