AB Key WS 2.1-2.3
Power, Product, Quotient

(2) $f(x) = x^{-1} - 3x^{-2} + 4x^{-3}$

$\boxed{f'(x) = \frac{-1}{x^2} + \frac{6}{x^3} - \frac{12}{x^4}}$ $\boxed{f'(1) = -7}$

(4) $\frac{8}{3}x^{-2} = y$

$\boxed{y' = \frac{-16}{3x^3}}$

(6) $y = 6x^{1/2}$ OR

$\boxed{y' = 3x^{-1/2}} = \boxed{\frac{3}{\sqrt{x}} = f'(x)}$

(8) $y = \frac{x}{2} - 3 + \frac{1}{x}$

$\boxed{y' = \frac{1}{2} - \frac{1}{x^2}} = \boxed{\frac{x^2 - 2}{2x^2}}$

(10) $y = x^4 - \frac{3}{2}x^3 + 5x^2 - 6x - 2$

$\boxed{y' = 4x^3 - \frac{9}{2}x^2 + 10x - 6}$

(12) $y = 2x^3 + 7x^2 - 4x$

$\boxed{y' = 6x^2 + 14x - 4}$

(14) $y = x^{1/3} - x^{2/3}$

$\boxed{y' = \frac{1}{3x^{2/3}} - \frac{2}{3x^{1/3}}}$ or $\boxed{\frac{1}{3\sqrt[3]{x^2}} - \frac{2}{3\sqrt[3]{x}}}$

(16) $y' = (x^2 - 4x - 6)(3x^2 - 10x - 3) + (x^3 - 5x^2 - 3x)(2x - 4)$

$\boxed{y' = 5x^4 - 36x^3 + 33x^2 + 84x + 18}$

$3x^4 - 10x^3 - 3x^2$
$-12x^3 + 40x^2 + 12x$
$-18x^2 + 60x + 18$
$2x^4 - 10x^3 - 6x^2$
$-4x^3 + 20x^2 + 12x$

(18) $y = \dfrac{x^2-4x-2}{x^2-1}$

$y' = (x^2-1)(2x-4) - (x^2-4x-2)(2x)$

$= \cancel{2x^3} - 4x^2 - 2x + 4 - \cancel{2x^3} + 8x^2 + 4x$

$\boxed{= \dfrac{4x^2+2x+4}{(x^2-1)^2}}$

(20) $y = \dfrac{x^2-x+1}{x^{1/3}}$

$y' = \dfrac{x^{1/3}(2x-1) - (x^2-x+1)\frac{1}{3x^{2/3}}}{(x^{1/3})^2}$

$y' = \dfrac{3x(2x-1) - (x^2-x+1)}{x^{2/3}(3x^{2/3})} = \dfrac{6x^2-3x-x^2+x-1}{3x^{4/3}}$

$= \boxed{\dfrac{5x^2-2x-1}{3x^{4/3}}}$

(26) $f(x) = (x-2)(x^2-3x-1)$

$f'(x) = (x-2)(2x-3) + (x^2-3x-1)(1)$

$= 2x^2 - 7x + 6 + x^2 - 3x - 1$

$= 3x^2 - 10x + 5$

$f'(-1) = 3 + 10 + 5 = 18$

$y + 9 = 18(x+1)$ $\boxed{y = 18x + 9}$

(30) $f(x) = \dfrac{4x}{x^2+4}$

$f'(x) = \dfrac{(x^2+4)4 - 4x(2x)}{(x^2+4)^2} = \dfrac{4x^2+16-8x^2}{(x^2+4)^2} = \dfrac{-4x^2+16}{(x^2+4)^2}$

$-4(x^2-4) = 0$

Horiz. tang. at $x = \pm 2$

Points: $(2, 1)$ $f(2)$

$(-2, -1)$ $f(-2)$

(22) $y = \dfrac{x-1}{x^2+2x+2}$

$$y' = \frac{x^2+2x+2 - (x-1)(2x+2)}{(x^2+2x+2)^2}$$

$$= \frac{x^2+2x+2-2x^2+2}{(x^2+2x+2)^2} = \boxed{\frac{-x^2+2x+4}{(x^2+2x+2)^2}}$$

(24) $y = \dfrac{x^2-k^2}{x^2+k^2}$

$$y' = \frac{(x^2+k^2)(2x) - (x^2-k^2)2x}{(x^2+k^2)^2}$$

$$= \frac{2x^3+2xk^2-2x^3+2xk^2}{(x^2+k^2)^2} = \boxed{\frac{4xk^2}{(x^2+k^2)^2}}$$

(28) $y = \left(\dfrac{x+3}{x+1}\right)\dfrac{(4x+1)}{1} = \dfrac{4x^2+13x+3}{x+1}$ $\quad(-1/2, -5)$

$$y' = \frac{(x+1)(8x+13) - (4x^2+13x+3)(1)}{(x+1)^2}$$

$$= \frac{8x^2+13x+8x+13-4x^2-13x-3}{(x+1)^2} = \frac{4x^2+8x+10}{(x+1)^2}$$

$$y'(-\tfrac{1}{2}) = \frac{4\cdot\frac{1}{4}+8(-\frac{1}{2})+10}{(-\frac{1}{2})^2} = \frac{1-4+10}{\frac{1}{4}} = \frac{7}{1/4} = 28$$

$$\boxed{y+5 = 28(x+1/2)}$$

(32) $h(x) = 3 + 8f(x)$

$h'(x) = 8\,f'(x)$

$h'(4) = 8\,(3)$ ← from table

$= 24$

(34) $h(x) = \dfrac{f(x)}{g(x)}$

$$h'(x) = \frac{g(x)\,f'(x) - f(x)\,g'(x)}{(g(x))^2}$$

$$h'(4) = \frac{3\pi\,(3) - (-8)(4)}{(3\pi)^2} = \boxed{\frac{9\pi + 32}{9\pi^2}}$$

(36) $h(x) = \dfrac{f(x) + 2}{-3g(x)}$

$$h'(x) = \frac{-3g(x)\,f'(x) - (f(x)+2)(-3g'(x))}{(-3g(x))^2}$$

$$h'(4) = \frac{-3(3\pi)(3) - (-6)(-3)(4)}{(-3 \cdot 3\pi)^2}$$

$$= \frac{-9\pi\,(\cancel{3}) - 24(\cancel{3})}{\underset{27}{\cancel{81}}\,\pi^2} = \frac{-9\pi - 24}{27\pi^2}$$

(38) $f(x) = \dfrac{x}{x-4}$

$$f'(x) = \frac{(x-4)(1) - (x)(1)}{(x-4)^2} = \frac{-4}{(x-4)^2} = \frac{-4}{x^2 - 8x + 16}$$

$$f''(x) = \frac{(x-4)^2(0) - (-4)(2x-8)}{(x-4)^4}$$

$$= \frac{8(x-4)}{(x-4)^4} = \boxed{\frac{8}{(x-4)^3}}$$

(40) tangent to $f(x) = x^2 - 6x + 7$

$\perp$ to $y = 2x + 4$

$f'(x) = 2x - 6$ $\quad\quad$ $\perp$ slope : $-\frac{1}{2}$

at the point of tangency the 2 slopes have to be the same!

$\therefore \quad 2x - 6 = -\frac{1}{2}$

$4x - 12 = -1$

$4x = 11$

$x = \frac{11}{4}$ $\quad\quad$ $f\left(\frac{11}{4}\right) = y = \frac{121}{16} - \frac{66}{4} + 7$

$= \frac{121}{16} - \frac{264}{16} + \frac{112}{16}$

$= -\frac{31}{16}$

We know:

1) $m = -\frac{1}{2}$

2) $\left(\frac{11}{4}, -\frac{31}{16}\right)$

$y + \frac{31}{16} = -\frac{1}{2}\left(x - \frac{11}{4}\right)$

$y + \frac{31}{16} = -\frac{1}{2}x + \frac{11}{8}$

$\boxed{y = -\frac{1}{2}x - \frac{9}{16}}$