Properties of Rational Exponents

Let a and b be real numbers and let m and n be rational numbers. The following properties have the same names as those listed on page 330, but now apply to rational exponents as illustrated.

	Property	Example
1.	$a^m \cdot a^n = a^{m+n}$	$5^{1/2} \cdot 5^{3/2} = 5^{(1/2+3/2)} = 5^2 = 25$
2.	$(a^m)^n = a^{mn}$	$(3^{5/2})^2 = 3^{(5/2 \cdot 2)} = 3^5 = 243$
3.	$(ab)^m = a^m b^m$	$(16 \cdot 9)^{1/2} = 16^{1/2} \cdot 9^{1/2} = 4 \cdot 3 = 12$
4.	$a^{-m} = \frac{1}{a^m}, a \neq 0$	$36^{-1/2} = \frac{1}{36^{1/2}} = \frac{1}{6}$
5.	$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$	$\frac{4^{5/2}}{4^{1/2}} = 4^{(5/2-1/2)} = 4^2 = 16$
6.	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$	$\left(\frac{27}{64}\right)^{1/3} = \frac{27^{1/3}}{64^{1/3}} = \frac{3}{4}$

Use properties of exponents to simplify the following expressions.

a. $7^{1/4} \cdot 7^{1/2} =$

b. $(6^{1/2} \cdot 4^{1/3})^2 =$

c. $(4^5 \cdot 3^5)^{-1/5} =$

d. $\frac{5}{5^{1/3}} = \frac{5^1}{5^{1/3}} =$

e. $\left(\dfrac{42^{1/3}}{6^{1/3}}\right)^2 =$

f. $\left(\sqrt[3]{x^2} \bullet \sqrt[6]{x^4}\right)^{-3}$

g. $\dfrac{\sqrt[3]{x} \bullet \sqrt{x^5}}{\sqrt{25x^{16}}}$

h. $12^{1/8} \bullet 12^{5/6} =$

i. $(5^{1/3} \bullet x^{1/4})^3 =$

j. $(2^6 \bullet 4^6)^{-1/6} =$

k. $\dfrac{10}{10^{2/5}} =$

l. $\left(\dfrac{56^{1/4}}{7^{1/4}}\right)^5$

RULE: $\sqrt{x} = x^{1/2}$ $\sqrt[3]{x} = x^{1/3}$ $\sqrt[4]{x} = x^{1/4}$ $\sqrt[n]{x} = x^{1/n}$

EXAMPLES: $8^{1/3} = \sqrt[3]{8} = 2$ $125^{1/3} = \sqrt[3]{125} = 5$

Evaluate each of the following without the use of a calculator!

1. $100^{1/2} =$	2. $16^{1/4} =$	3. $100,000^{1/5} =$	4. $27^{1/3} =$
5. $81^{1/2} =$	6. $216^{1/3} =$	7. $144^{1/2} =$	8. $1^{1/4} =$
9. $225^{1/2} =$	10. $49^{1/2} =$	11. $1,000^{1/3} =$	12. $25^{1/2} =$

RULE: $x^{3/2} = \left(x^{1/2}\right)^3 = \left(\sqrt{x}\right)^3$ $x^{m/n} = \left(\sqrt[n]{x}\right)^m$

EXAMPLES: $8^{2/3} = \left(8^{1/3}\right)^2 = \left(\sqrt[3]{8}\right)^2 = (2)^2 = 4$ $25^{3/2} = \left(\sqrt{25}\right)^3 = (5)^3 = 125$

Evaluate each of the following without the use of a calculator!

1. $100^{3/2} =$	2. $16^{3/4} =$	3. $1000^{2/3} =$	4. $25^{3/2} =$
5. $8^{4/3} =$	6. $64^{2/3} =$	7. $64^{3/2} =$	8. $81^{1/2} =$
9. $625^{3/4} =$	10. $49^{3/2} =$	11. $32^{3/5} =$	12. $121^{-1/2} =$

A negative exponent was slipped into that last problem! How did you deal with it?

RULE:	$x^{-2} = \frac{1}{x^2}$	$x^{-5} = \frac{1}{x^5}$	$x^{-n} = \frac{1}{x^n}$
EXAMPLES:	$8^{-2} = \frac{1}{8^2} = \frac{1}{64}$	$25^{-3/2} = \left(\sqrt{25}\right)^{-3} = (5)^{-3} = \frac{1}{5^3} = \frac{1}{125}$	

Evaluate each of the following without the use of a calculator!

1. $10^{-2} =$	2. $16^{-1/2} =$	3. $1000^{-2/3} =$	4. $5^{-2} =$
5. $125^{-2/3} =$	6. $\left(\frac{1}{4}\right)^{-1/2} =$	7. $49^{-1/2} =$	8. $81^{-1/2} =$
9. $6^{-3} =$	10. $32^{-3/5} =$	11. $7^{-2} =$	12. $\left(\frac{9}{16}\right)^{-1/2} =$