WRITING EXPONENTIAL FUNCTIONS

The student is expected to write exponential functions in the form $f(x) = ab^x$ (where b is a rational number) to describe problems arising from mathematical and real-world situations, including growth and decay.

TELL ME MORE...

Exponential functions are based on relationships involving a constant multiplier. You can write an exponential function in general form. In this form, a represents an initial value or amount, and b, the constant multiplier, is a growth factor or factor of decay.

$$f(x) = ab^x$$

Exponential Growth

If $b > 1$, then the starting amount, a, is multiplied by a number greater than 1 and will increase as x increases.

The value of the dependent variable grows as the value of the independent variable increases.

Exponential Decay

If $0 < b < 1$, then the starting amount, a, is multiplied by a number less than 1 and will decrease as x increases.

The value of the dependent variable decays as the value of the independent variable increases.

If you know the starting point, a, and constant multiplier, b, then you can write the exponential function in general form. The values of a and b can be deduced from a table, verbal situation, or diagram. In a table, look for the ratio of successive values of the dependent variable. If these ratios are constant, then the function is an exponential function. The constant ratio is equivalent to the constant multiplier used to generate the exponential function.

EXAMPLES

EXAMPLE 1: The table contains some points on the graph of an exponential function. Based on the table, what function represents the same relationship?

x	y
0	0.04
1	0.2
2	1
3	5
4	25

STEP 1 Determine the constant multiplier, or constant ratio, between successive function values. This multiplier will be the value of b.

x	y	
0	0.04	
1	0.2	$0.2 \div 0.04 = 5$
2	1	$1 \div 0.2 = 5$
3	5	$5 \div 1 = 5$
4	25	$25 \div 5 = 5$

$b = 5$ **b is the common ratio**

STEP 2 Identify the initial value, or the function value when $x = 0$. This value will be the value of a.

$a = 0.04$ **a is the initial value at 0**

STEP 3 Use the values of a and b that you determined in the table to write the function in $y = ab^x$ form.

$a = 0.04$ and $b = 5$, so $y = 0.04(5)^x$.

x	y
0	0.04
1	0.2
2	1
3	5
4	25

EXAMPLE 2: In January, the Parks and Wildlife Department released 267 bass into a newly constructed pond. Each month, the population of bass in the pond increases by 4.2%. At this rate of growth, what function can be used to determine the population of bass m months after January?

STEP 1 Determine the growth rate or rate of decay. This rate will be the value of b.

Each month, the population increases by 4.2%, so the growth rate is 100% + 4.2%, or 104.2% each year. Percents should be written as their decimal equivalents.

$b = 1.042$

STEP 2 Identify the initial value, or number of bass at the beginning. This value will be the value of a.

The Parks and Wildlife Department released 267 bass into a pond with no fish. The initial amount of bass is 267.

$a = 267$

STEP 3 Use the values of a and b that you interpreted from the situation to write the function in $p(m) = ab^m$ form. Let m represent the number of months and $p(m)$ represent the population as a function of m.

$a = 267$ and $b = 1.042$, so

$p(m) = 267(1.042)^m$.

YOU TRY IT!

The table contains some points on the graph of an exponential function. Based on the table, what function represents the same relationship?

x	y
0	64
1	48
2	36
3	27
4	20.25

Value of a: _____ b: _____

Function: _____

EXAMPLE 3: A pyramid at Chichén Itzá in Mexico contains several layers that are in the shape of square prisms. The first layer has a side length of 50 meters and each successive layer has a side length that is 95% of the one directly below it. What function can be used to find the side length, in meters, of layer L, where $1 \leq L \leq 6$?

El Osario Pyramid, Chichén Itzá, Mexico

STEP 1 Determine the growth rate or rate of decay. This rate will be the value of b.

Each successive layer has a side length that is 95% of the one directly below it, so the rate of decay is 95%. In an equation, percents should be written as their decimal equivalents.

$\boldsymbol{b = 0.95}$

STEP 2 Identify the initial value of the dependent variable, the side length of the first layer. This value will be the value of a.

The ground layer is the first layer. The side length of the ground layer is the initial value, 50 meters.

$\boldsymbol{a = 50}$

STEP 3 Use the values of a and b that you interpreted from the situation to write the function in $s(L) = ab^L$ form. Let L represent the number of layers above the base layer and $s(L)$ represent the side length as a function of L.

$a = 50$ and $b = 0.95$, so $\boldsymbol{s(L) = 50(0.95)^L}$.

MAKE A NOTE . . .

How can you tell if a problem involves exponential growth or exponential decay? Record your thoughts here.

PRACTICE

For questions 1-6, determine if the data represent exponential growth or exponential decay. Then, write the function in $f(x) = ab^x$ form.

1.

x	$g(x)$
0	100
1	10
2	1
3	0.1
4	0.01

2.

x	$h(x)$
0	2
1	3
2	$4\frac{1}{2}$
3	$6\frac{3}{4}$
4	$10\frac{1}{8}$

3.

x	$b(x)$
1	20
2	200
3	2000
4	20,000
5	200,000

4.

m	$j(m)$
0	1.2
1	0.6
2	0.3
3	0.15
4	0.075

5.

t	$p(t)$
0	210
1	84
2	33.6
3	13.44
4	5.376

6.

r	$D(r)$
1	6
2	36
3	216
4	1296
5	7776

For each situation below, write an exponential function that can be used to describe the situation.

7. An official NBA basketball must be inflated so that when it is dropped, it must not bounce back more than 75% of the height from which it was dropped. If a basketball is dropped from a height of 72 inches, what function, $r(b)$, describes the height of the basketball as a function of b, the number of bounces?

8. The population of Bexar County, Texas, for a few recent years is shown in the table.

Year	Year Since 2010	Population (millions)
2010	0	1.723
2011	1	1.756
2012	2	1.789
2013	3	1.822
2014	4	1.856

If t represents the number of years since 2010 and $P(t)$ represents the population in millions, what function, $P(t)$ best describes the data in the table?

9. A chiropterologist is a scientist who studies bats. A bat colony is discovered and a chiropterologist calculates that there are 2,100 bats in the colony. If the population of the colony doubles each year, which function, $B(t)$, describes the population of the colony t years after discovery?

A $B(t) = 2100\left(\frac{1}{2}\right)^t$

B $B(t) = 2100(2)^t$

C $B(t) = 2100 + (2)^t$

D $B(t) = 2100 + \left(\frac{1}{2}\right)^t$

10. Chuy purchased a used truck for $11,500. According to an online vehicle website, his truck will depreciate, or lose value, at a rate of 5.5% each year. What function, $d(x)$, represents the value of Chuy's truck x years after its purchase?

F $d(x) = 11{,}500(0.945)^x$

G $d(x) = 11{,}500(1.055)^x$

H $d(x) = 11{,}500(0.055)^x$

J $d(x) = 11{,}500(5.5)^x$