Graphical Behavior of Functions

As part of exploring how functions change, it is interesting to explore the graphical behavior of functions.

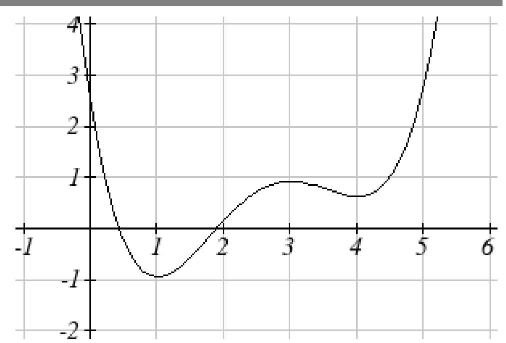
Increasing/Decreasing

A function is **increasing** on an interval if the function values increase as the inputs increase. More formally, a function is increasing if $f(b) > f(a)$ for any two input values a and b in the interval with $b>a$. The average rate of change of an increasing function is **positive.**

A function is **decreasing** on an interval if the function values decrease as the inputs increase. More formally, a function is decreasing if $f(b) < f(a)$ for any two input values a and b in the interval with $b>a$. The average rate of change of a decreasing function is **negative.**

Example 7

Given the function $p(t)$ graphed here, on what intervals does the function appear to be increasing?



Notice: We use open intervals (intervals that don't include the endpoints) since the function is neither increasing nor decreasing at t = 1, 3, or 4.

Local Extrema

A point where a function changes from increasing to decreasing is called a **local maximum**.

A point where a function changes from decreasing to increasing is called a **local minimum**.

Together, local maxima and minima are called the **local extrema**, or local extreme values, of the function.

Example 8

Using the cost of gasoline function from the beginning of the section, find an interval on which the function appears to be decreasing. Estimate any local extrema using the table.

t	2	3	4	5	6	7	8	9
$C(t)$	1.47	1.69	1.94	2.30	2.51	2.64	3.01	2.14

Example 9

Use a graph to estimate the local extrema of the function $f(x) = \frac{2}{x} + \frac{x}{3}$. Use these to determine the intervals on which the function is increasing.

NOW use your graphing calculators to determine the exact local minimum and local maximum.